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Three essays on financial economics

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The student author, whose presentation of the scholarship herein was approved by the program of study committee, is solely responsible for the content of this dissertation. The Graduate College will ensure this dissertation is globally accessible and will not permit alterations after a degree is conferred.

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DEDICATION

I would like to dedicate this dissertation to my wife Jiuyi Deng, to my parents Lin Ma and Wensong Deng, without whose support I would not have been able to complete this work.

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ABSTRACT

The main subjective of this dissertation is to analyze three issues of current interest in financial economics. Chapter 2 shows that the presence of the initial short position will give traders an incentive to manipulate by buying less or selling more. When the initial short position is not revealed, this distortion will mislead the firm through the performance of the stock in the financial market. In this circumstance, the firm may mistakenly reject some good projects due to the information asymmetry. After the revealing of the initial short position, the information asymmetry could be eliminated, and thus improve the financial market's efficiency potentially. Chapter 3 studies how strategic risk among investors can help explain both underpricing and underreaction in initial public offerings (IPOs) by using theoretical and simulation tools. If the IPO raises more capital for the firm, the post-IPO value of a firm will be higher. Hence an IPO subscriber faces strategic risk: the value of subscribing depends on the aggregate subscription rate. As this risk is resolved immediately after the IPO, the IPO itself is underpriced. Moreover, since individual investors have limited wealth, a higher offer price raises the risk of undersubscription. Investors respond by demanding a larger discount: the offer price appears to underreact to public news. Chapter 4 develops a theoretical model supported by empirical evidence examining the relation between brokerage choice and market strength. Our model shows that although internal transactions have the potential side benefits of higher commission and lower search costs to an agent, in a strong housing market, most brokerage firms still prefer external transactions because of the greater demand for housing. However, when the market weakens, external demand for housing decreases, and brokerage firms become more willing to engage in internal transactions. This occurs at the expense of lowering the selling price, which speaks to a principal-agent incentive misalignment problem.

CHAPTER 1. OVERVIEW

This dissertation consists of three essays discussing topics on financial economics. The first essay seeks to present a case where price failed to efficiently allocate resources on the stock market. It is commonly believed that prices play an important role in allocating scarce resources because they convey information that improves the efficiency in the allocation of scarce resources. Specifically, in secondary financial markets, the prices may reflect certain information that can increase the efficiency of real investment decisions. However, some researches pointed out that there are limitations in the allocative role of prices in financial markets. In the first essay, we set up a model for short selling trading strategy and examine its consequences. The results of the model indicate that this trading strategy will indeed cause inefficiency. We show that prices may fail to efficiently allocate resources when the potentially informed speculator has initial short position on the asset. And we suggests that revealing the short position will help alleviate this problem, i.e., eliminating the information asymmetry will improve the efficiency of the financial market.

The second essay attempts to explain how strategic risk can lead to IPO underpricing and underreaction. Intuitively, shares sold in an IPO are more valuable if the firm reaps more revenue from the IPO. But individual agents have limited wealth, so an IPO requires the participation of multiple investors. As a result, investors in an IPO face strategic risk: the value of the shares depends on the number of others who choose to subscribe to the IPO, which cannot be exactly predicted *ex ante*. Since an agent's reservation price is the price at which she is just willing to subscribe to the IPO. But if, given her information about the IPO, she is indeed just willing to subscribe, then she knows that some others are likely to have received slightly more negative information than her own and thus will choose not to subscribe. Hence, her reservation price reflects a positive probability of undersubscription, which - if it occurs - will lower the firm's value. Also the same mechanism yields underreaction to public information that is observed prior to the IPO. Good news leads the firm to raise the IPO price. As agents' wealth is limited, the risk of undersubscription is now greater: investors face even more strategic risk. Hence the IPO must be even more underpriced than before the good news was received.

The third essay tries to answer the following questions on the real estate market. When do agents prefer to engage in external versus internal transactions? How do internal transactions, and in particular dual agent transactions, affect sale price? Do these brokerage choice change depending on the strength of the housing market? Our study attempts to examine these questions from a new perspective. Specifically, how will the preference for brokerage type change when market strength changes. Moreover, after controlling for market strength, what happens to home prices in internal versus external transactions. The key findings in the third essay indicate two important results. First, a potential self-correction mechanism for the principal-agent problem may exist within the housing market. As the market strengthens, external buying orders become more attractive to agents. Leading them to engage in more external transactions. Second, when the market weakens, internal transactions increase. The increase in internal transactions further reduces market price which drives sellers out and further reduces the strength of the market. Hence, the equilibrium brokerage choice creates a self-reinforcing mechanism toward generating more extreme market conditions.

CHAPTER 2. SHORT AND DISTORT

It is widely studied that prices in the financial market provide information that help the firm to improve efficiency of its investment decisions. In this paper, we incorporate an initial short position in our model, and show that the initial short position will give traders an incentive to manipulate by buying less or selling more. When the initial short position is not revealed, this distortion will mislead the firm through the performance of the stock in the financial market. In this circumstance, the firm may mistakenly reject some good projects due to the information asymmetry. After the revealing of the initial short position, the information asymmetry could be eliminated, and thus improve the financial market's efficiency potentially. So We propose policies including revealing of the short position to help the financial market to fulfill its function.

2.1 Introduction

The activities of short-selling in capital market drew a lot of attentions by academics, regulators, and politicians in recent years. As is pointed out by (Boehmer, Jones, and Zhang, 2008), Short seller account for more than 20% of trading volume and are generally regarded as traders with access to value-relevant information, therefore it is widely believed that short selling activity is an essential part of the price discovery mechanism. However, the consequences of short-selling in capital market are controversial. Some advocates argue that short-selling activities benefit capital market in various ways. For example, IOSCO (International Organization of Securities Commissions) says: Short selling plays an important role in capital markets for a variety of reasons, including more efficient price discovery, mitigating price bubbles, increasing market liquidity, facilitating hedging and other risk management activities. (Andrew Baker, "Why short selling is good for capital markets", Financial Times, FEBRUARY 20, 2011). But some critics claim that short-selling may lead to market downturns and may be unethically used by traders to make profit. For example, Elvis Picardo pointed out an unethical trade strategy called "short and distort" in an article: this technique takes place when traders manipulate stock prices in a bear market by taking short positions and then using a smear campaign to drive down the target stocks. Figure 2.1 depicts the "short and distort" manipulation. This is the mirror version of the pump and dump, where crooks buy stock (take a long position) and issue false information that causes the target stock's price to increase (Elvis Picardo,"Ethics And The Role Of Short Selling", Investopedia). In this paper, we will set up a model for this trading strategy and examine its consequences. The results of this model indicate that this trading strategy will indeed cause inefficiency. How can we improve this inefficiency? Simply outlawing short-selling may not be a good choice since short-selling may have some positive influences such as increasing market liquidity. Instead, we propose that the regulator can reduce the inefficiency by requiring large short-sellers to reveal their short positions. In practice, short interest information is available, although the information is usually delayed. However, it is the large speculator's short position instead of the total short interest that will give the firm information to eliminate the inefficiency.

From another point of view, this paper seeks to present a case where price failed to efficiently allocate resources. As is proposed by (Hayek, 1945), prices play an important role in allocating scarce resources because they convey information that improves the efficiency in the allocation of scarce resources. Specifically, in secondary financial markets, the prices may reflect certain information that can increase the efficiency of real investment decisions. Several studies have been made regarding this topic, for example, in Khanna et al.(1994) adn Leland (1992), firms use information inferred from stock price levels to make firm capacity choice. In Subrahmanyam and Titman (2001), stock prices have an impact on firm cash flows because stakeholders such as employees, suppliers and customers condition on price levels when deciding whether to stay with the firm or leave.

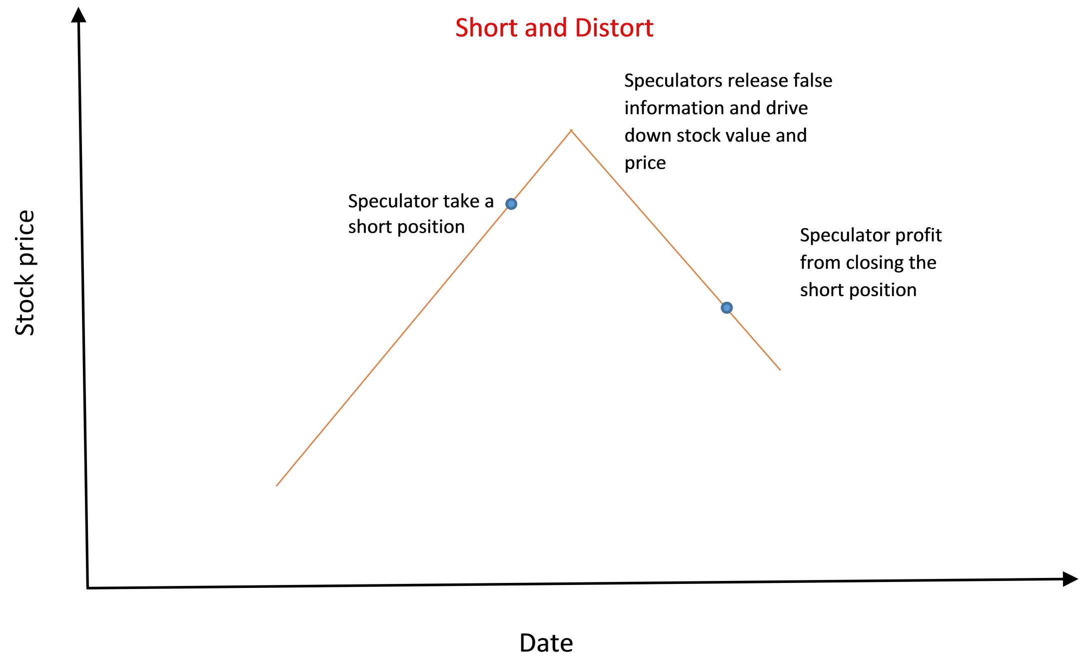

Figure 2.1: Short and Distort Manipulation

The basic argument for the allocation role of price is as follows: in the financial market, speculators tend to trade on their own information, incorporating it into prices and eliminating any mis-pricing. For example, if speculators have negative private information about a stock, they will find it profitable to sell the stock. This action will push down the price, reflecting the speculators' information. If prices are informative, it is natural to expect firms to use the information in prices to make decisions that may increase firm value (such as investment). This is the feedback effect of prices.

However, some researches pointed out that there are limitations in the allocative role of prices in financial markets. Goldstein and Guembel (2008) analyzes how the feedback effect of prices provides an incentive for an uninformed speculator to manipulate the stock prices by short-selling the stock. This reduces the stock prices and lead the firm to make incorrect investment decision, thus generating a profit on the speculator's short position. When such manipulation occurs, the information conveyed by prices is misleading, and this distorts resource allocation will reduce the economic efficiency. Khanna and Sonti (2004) shows that feedback from prices to asset value can generate herding. Assuming that a sequence of buy orders increases firm value in the good state of the world, they show that a late trader with an inventory of the stock will buy after receiving a negative private signal and observing a sequence of buy orders. Following previous trades, this trader believes that the state of the world is likely to be good and buys to increase the value of his inventory.

In this paper, we will show that prices may fail to efficiently allocate resources when the potentially informed speculator has initial short position on the asset. And we suggests that revealing the short position will help alleviate this problem. The basic setting of the model is as follows. There is a firm that faces an investment opportunity with uncertain net present value (NPV). There is a speculator who may or may not have the information about the profitability of the investment. The profitability of the project is relevant to the optimal investment decision but is not yet known to the firm. The speculator will optimally choose to trade in the firm's stock based on her information. The trading process is modeled in a market micro-structure setting based on Kyle (1985). The information of the speculator will then get partially reflected in the stock price. Thus the firm could take the information conveyed by the price into consideration of its own investment decisions. In this paper, we assume that the speculator initially has a short position which she needs to pay back, but firm and market do not know about it. In this case, the prices in market will mislead investment decisions and cause inefficiency. There are two main difference between this paper and Goldstein and Guembel (2008). First, we impose a continuous short position in this model instead of a discrete position in Goldstein and Guembel (2008). Second, unlike GG model, we study the case in which participants other than the speculator are unaware of the short position. This lets us study the welfare effects of requiring disclosure. And by comparing the benefit before and after the revealing of the short position we will have a policy implication in the financial market.

Our main result is:

For the cases in which participants other than the speculator are unaware of the short position, we have

1) If the speculator has no initial position, then she will sell if she is negatively informed; will buy if she is positively informed; and will mix between selling and doing nothing if she is uninformed.

2) If the speculator has a small initial short position, she will sell if she is negatively informed or uninformed; and will buy if she is positively informed.

3) If the speculator has a large initial short position, then she will always sell no matter she is negatively informed, positively informed or uninformed. This is the case that market failure causes inefficiency in the resource allocation. The firm could not receive any information through the financial market, and will make wrongly investment decisions (reject the project when it is actually worth investing) if the short position is not revealed.

For the cases in which participants other than the speculator are aware of the short position, we can see that the firm may also choose to invest when the trading quantity is low. The market efficiency has been improved from two aspects. First, the behavior of the positively informed speculator will be less aggressive when the initial short position is large. This is because the positively informed speculator could not pretend to be negatively informed without any cost. Second, from the firm's point of view, a lower trading quantity may not entirely represent a bad signal, it might also conducted by the needs of closing initial short position from the positively informed speculator. Even if when the initial short position is very large, which makes all types speculators sell for sure, the firm manager will put her own judgment into consideration and try not to reject the project easily like before.

Thus by comparing the different cases before and after the revealing of the short position, we suggest a policy to improve the inefficiency, which may help to justify the proposal that large short position for some agents should be revealed in the market.

2.2 The Model

In this section, I will introduce the basic settings and some general results

2.2.1 Basic Settings

The model has three dates $t \in \{0, 1, 2\}$ and a firm whose stock is traded in the financial market. The firm's manager needs to make an investment decision. In $t = 0$, a risk-neutral speculator may or may not know whether the investment is profitable or not. The speculator has an initial short position which the firm and market do not know. The short position is measured by s which is the amount of stock she needs to pay back. Trading in the financial market occurs in $t = 1$. In addition to the speculator, two other types of agents participate in the financial market: noise trader and a risk-neutral market maker. The latter collects the orders from the speculator and the noise trader and sets a price at which she executes the orders out of her inventory. The information of the speculator may get reflected in the price via the trading process. Speculator has to close her short position at the end of period 1. In $t = 2$, the manager makes the investment decision, which may be affected by the stock price realizations. Finally, all uncertainty is realized and pay-offs are made. Figure 2.2 shows the time line of the short sale model.

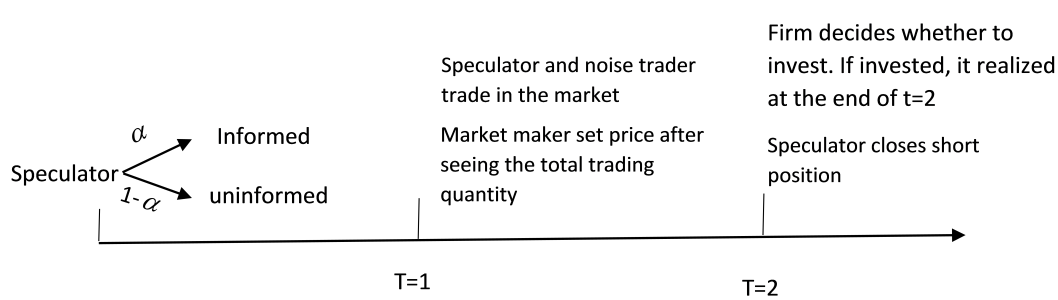

Figure 2.2: Timeline of the Short Sale Model

In $t = 0$, the speculator receives a perfectly informative private signal $\omega \in \{h, l, \emptyset\}$. If ω is h, which occurs with probability $\frac{\alpha}{2}$, she knows that the project is profitable; if ω is l, which also occurs with probability $\frac{\alpha}{2}$, she knows that the project is unprofitable; and if ω is $\emptyset$, which occurs with probability $1 - \alpha$, she believes that the project is either profitable or unprofitable with equal probabilities. We will sometimes use the term positively informed speculator to refer to the speculator when she obtains the signal h. We will analogously use the terms negatively informed speculator and uninformed speculator.

Suppose that the firm has an investment opportunity that requires a fixed investment. The firm's manager acts in the interest of shareholders and chooses whether or not to invest with the objective to maximize the expected firm value. The firm faces uncertainty over the quality of the available investment opportunity. We denote the value of the investment if it is profitable as V^+; the value of the investment if it is unprofitable as V^-. We assume that it is worth investing if the project is profitable, but not when the project is unprofitable: $V^+ > 0 > V^-$. Besides, following Goldstein and Guembel (2008) we assume

$$\bar{V} = \frac{V^+ + V^-}{2} > 0$$

and

$$(1 - \alpha)\,\bar{V} + \frac{\alpha}{2} V^- < 0$$

The first inequality implies that the ex-ante NPV of the project is positive, this restriction is imposed so that without further information, the manager will choose to take the investment. The second inequality implies that the probability α of informed trader is sufficiently high so that the firm optimally rejects the project after orders that do not distinguish between the negatively informed and the uninformed speculator. This restriction is imposed so that the information content of a sell order is strong enough to justify the cancellation of the investment, even though it is known that the sell order could be generated by both the negatively informed and the uninformed speculator.

The decision of the firm can be conditioned on the information it has about the underlying profitability of the project. The firm may learn such information from the price of its equity in the financial market. The original price of the stock is V_0 in $t = 0$, we can normalize it to be zero so that the stock price equals the value of investment.

The speculator has a short position in $t = 0$, which the firm and market do not know about. In section 4, we assume that government is able to require the speculator to reveal her position. Depending on her signal, the speculator may wish to trade in the financial market in $t = 1$. In addition to the speculator and the market maker, there is a noise trader. Denoting the order of the noise trader as D we assume that $D = -1, 0, 1$ with equal probabilities; that is, the noise trader buys, sells, or does not trade with equal probabilities. For now, we treat the noise traders' orders as exogenous. Denoting the order of the speculator as D_S, $D_S \in \{-1, 0, 1\}$. That is, speculator can sell/not trade/buy one unit of the stock. Besides, the speculator needs to close her short position in $t = 1$, for simplicity, we assume that she can close the short position by paying back money at the end of period 1. The money she needs to pay is equal to the current value of the stock. This may be viewed as the speculator has signed a one-time swap with another agent, which requires the speculator to pay some money to the agent in $t = 1$ and the amount of the money is equal to the current value of the stock in short position.

Orders are submitted simultaneously to a market maker who sets the price and absorbs order flows out of her inventory. The market maker sets the price equal to expected asset value given the information contained in past and present order flows. This assumption is justified when the market making industry is competitive. The market maker can only observe total order flow $Q = D_S + D$, but not its individual components. Possible order flows are therefore $Q \in \{-2, -1, 0, 1, 2\}$. Figure 2.3 describes all the potential trading quantity results, there will be a price of the stock for each trading quantity. The price is a function of total order flow: $P_q = \pi_q E\left[V_\omega \left| Q = q\right.\right]$, where V_ω denotes the expected value of the project if it is implemented and the signal is ω, and π_Q denote the firm's probability of implementing the project when it observes the quantity Q. Assume the speculator submits market orders before the market price being set, that is, orders are not contingent on current price. Thus, the speculator's order will be contingent only on her own signal ω. Then, the market maker will set the price conditional on the information she has about the quantities Q traded by speculator and noise trader. The firm manager observes Q and will use it in her investment decisions. Firm will invest if $E\left[V_\omega \left| Q = q\right.\right] > 0$, will not invest if $E\left[V_\omega \left| Q = q\right.\right] < 0$, will mix between investing and not investing if $E\left[V_\omega \left| Q = q\right.\right] = 0$.

Q / D_s+D	-2	-1	0	1	2
	-1+(-1)	-1+0	-1+1	1+0	1+1
		0+(-1)	0+0	0+1	
			1+(-1)		

Figure 2.3: Potential Trading Quantities

We assume that:

(i) The speculator chooses $\{u(\omega)\}$ to maximize his expected final pay-off, given the price-setting rule, the strategy of the manager, and the information she has at the time she submits the trade;

(ii) The firm maximizes its expected value given its belief and all other strategies;

(iii) A price-setting strategy by the market maker $\{P_q\}$ that allows him to break even in expectation, given his belief and all other strategies.

2.2.2 Preliminaries

Assume that the trader starts with a short position s. Other participants may or may not be aware of this position. Let $\omega \in \{h, l, \varnothing\}$ denote the signal. In particular, the trader is uninformed ($\omega = \varnothing$) with probability $1 - \alpha$. The speculator knows that the project is profitable ($\omega = h$) with probability $\alpha/2$ and that the the project is unprofitable ($\omega = l$) with probability $\alpha/2$. Let p_ω^b, p_ω^s and p_ω^n be the trader's equilibrium probability of buying, selling, and doing nothing when the signal is $\omega \in \{h, l, \varnothing\}$. (Of course, $p^b_\omega + p^s_\omega + p^n_\omega = 1$ for each signal ω.)

Claim 2.1. *The following property holds.*

Single Crossing Property (SCP). *In any perfect Bayesian equilibrium, if $p^b_l > 0$, then $p^b_\varnothing = 1$. And if $p^b_\varnothing > 0$, then $p^b_h = 1$. Likewise, if $p^s_h > 0$ then $p^s_\varnothing = 1$; and if $p^s_\varnothing > 0$ then $p^s_l = 1$.*

Proof. Let $D \in \{-1, 0, 1\}$ be the noise trader's (random) demand. Let V_ω denote the expected value of the project if it is implemented and the signal is ω. (Thus, $V_h = V^+$, $V_l = V^-$, and $V_\varnothing = \overline{V}$.) Let π_Q denote the firm's probability of implementing the project when it observes the quantity Q (whether or not Q occurs with positive probability in equilibrium). Since the trader borrowed s shares of the firm's stock, she must pay sP_{D+1} when her short position is settled. Hence, the trader's payoff is as follows as a function of the signal ω and the trader's action (whether or not this action is ever taken in equilibrium):

- Buy 1 share: she gets $U^b_\omega = V_\omega E_D[\pi_{D+1}] - (1+s) E_D[P_{D+1}]$.
- Do nothing: she gets $U^n_\omega = -sE_D[P_D]$.
- Sell 1 share: she gets $U^s_\omega = -V_\omega E_D[\pi_{D-1}] - (-1+s) E_D[P_{D-1}]$.

Let $D_S \in \{-1, 0, 1\}$ be the number of shares that the speculator buys. For any two actions $a, a' \in \{b, s, n\}$, let $\Delta^{a,a'}_\omega$ denote the trader's relative payoff $U^a_\omega - U^{a'}_\omega$ from playing a vs. a' when her type is ω. The relative payoff from buying 1 share vs. doing nothing is

$$\Delta^{b,n}_\omega = U^b_\omega - U^n_\omega = V_\omega E_D[\pi_{D+1}] - E_D[P_{D+1}] + s(E_D[P_D] - E_D[P_{D+1}]).$$

The relative payoff from doing nothing vs. selling 1 share is

$$\Delta^{n,s}_\omega = U^n_\omega - U^s_\omega = V_\omega E_D[\pi_{D-1}] - E_D[P_{D-1}] + s(E_D[P_{D-1}] - E_D[P_D]).$$

Finally, the relative payoff $\Delta_\omega^{b,s}$ from buying vs. selling is just the sum of the relative payoff $\Delta_\omega^{b,n}$ of buying vs. doing nothing, plus the relative payoff $\Delta_\omega^{n,s}$ from doing nothing vs. selling. For each D_S in $\{-1,0,1\}$, $E_D\left[\pi_{D+D_S}\right] \geq 1/3$ since the firm invests for sure if $Q=0$ which occurs with probability $1/3$ for each such D_S. Thus,

$$\Delta_h^{b,n} > \Delta_\varnothing^{b,n} > \Delta_l^{b,n} \text{ and } \Delta_h^{n,s} > \Delta_\varnothing^{n,s} > \Delta_l^{n,s}. \tag{2.1}$$

Equation (2.1) implies SCP. This is because, first suppose p_l^b is positive. Then $\Delta_l^{b,n}$ and $\Delta_l^{b,s} = \Delta_l^{b,n} + \Delta_l^{n,s}$ must be nonnegative. But then by (2.1), $\Delta_\varnothing^{b,n}$ and $\Delta_\varnothing^{b,s} = \Delta_\varnothing^{b,n} + \Delta_\varnothing^{n,s}$ must be positive: $p_\varnothing^b$ must equal one. The other parts of SCP are proved analogously. □

Corollary 2.1. *In any perfect Bayesian equilibrium, if $p_h^b < 1$, then $p_\varnothing^b = p_l^b = 0$. And if $p_l^s < 1$, then $p_\varnothing^s = p_h^s = 0$.*

Proof. By contrapositive of the Single Crossing Property (SCP). □

The following Lemma 2.1 provides a general strategy result of the firm and market maker when total order flow $Q=0$.

Lemma 2.1. *In any perfect Bayesian equilibrium, we will have π_0=1 and $P_0 = \bar{V}$, the firm will always invest when total order flow $Q=0$, and the market maker will set the price equal to $\bar{V}$.*

Proof. See Appendix. □

The intuition behind Lemma 1 is straightforward. Since whatever the speculator chooses to do, buying, selling or doing nothing, there will always be a probability of $\frac{1}{3}$ that the total order flow Q appears to be 0 due to the existence of the noise trader. So no matter what type the speculator is, Q=0 will not deliver any information to the firm and market maker. Notice that by assumption the ex-ante NPV of the project is positive, which means the firm manager will choose to take the investment without further information. Thus the firm will always invest when total order flow $Q=0$, and based on the same information, the market maker will set the price equal to $\bar{V}$, the average of the profitable investment value and the unprofitable investment value.

Claim 2.2. *In any perfect Bayesian equilibrium, if all types sell or no types buy, then the firm and market maker will believe the deviator is positively informed; and if all types buy or no types sell, then the firm and market maker will believe the deviator is negatively informed;*

Proof. First we will introduce a concept in the signaling game called Condition D1 (Sobel, Joel. (2007)).

Condition D1: An equilibrium refinement that requires out-of-equilibrium beliefs to be supported on types that have the most to gain from deviating from a fixed equilibrium.

Notice that according to the proof of Single Crossing Property, we have

$$\Delta_\omega^{b,n} \geq 0 \Leftrightarrow V_\omega \geq \frac{(1+s)\,E_D P_{D+1} - sE_D P_D}{E_D\left(\pi_{D+1}\right)}$$

$$\Delta_\omega^{n,s} \geq 0 \Leftrightarrow V_\omega \geq \frac{(1-s)\,E_D P_{D-1} + sE_D P_D}{E_D\left(\pi_{D-1}\right)}$$

$$\Delta_\omega^{b,s} = \Delta_\omega^{b,n} + \Delta_\omega^{n,s} \geq 0 \Leftrightarrow V_\omega \geq \frac{(1+s)\,E_D P_{D+1} + (1-s)\,E_D P_{D-1}}{E_D\left(\pi_{D+1}\right) + E_D\left(\pi_{D-1}\right)}$$

So if all types sell or no types buy, then the set of responses $((P_q, \pi_q)_{q=-2}^2)$ to a deviation (to a higher action), which makes that deviation profitable, is larger for higher types.

Thus D1 implies that firm and market maker will believe the deviator is positively informed.

Similarly, we have the symmetric results as follow

$$\Delta_\omega^{b,n} \leq 0 \Leftrightarrow V_\omega \leq \frac{(1+s)\,E_D P_{D+1} - sE_D P_D}{E_D\left(\pi_{D+1}\right)}$$

$$\Delta_\omega^{n,s} \leq 0 \Leftrightarrow V_\omega \leq \frac{(1-s)\,E_D P_{D-1} + sE_D P_D}{E_D\left(\pi_{D-1}\right)}$$

$$\Delta_\omega^{b,s} = \Delta_\omega^{b,n} + \Delta_\omega^{n,s} \leq 0 \Leftrightarrow V_\omega \leq \frac{(1+s)\,E_D P_{D+1} + (1-s)\,E_D P_{D-1}}{E_D\left(\pi_{D+1}\right) + E_D\left(\pi_{D-1}\right)}$$

So if all types buy or no types sell, then the set of responses $((P_q, \pi_q)_{q=-2}^2)$ to a deviation (to a lower action), which makes that deviation profitable, is larger for lower types.

Therefore D1 implies that firm and market maker will believe the deviator is negatively informed. □

The following several Lemmas together summarize the general strategies of the firm and market maker under different total order flow. These strategy results are independent of the speculator's initial position, so they will be held no matter the initial short position is revealed or not.

Lemma 2.2. *In any perfect Bayesian equilibrium, if* $p_h^b > \frac{2\bar{V}}{\alpha V^+}$*, then* $E\left(V_\omega \left| Q = -1\right.\right) < 0$*, the firm will not choose to invest when total order flow* $Q = -1$*, the market maker will set a price to equal zero,* $P_{-1} = 0$*.*

Proof. See Appendix. □

The intuition behind this Lemma is based on the Single Crossing Property we introduced before. If the negatively informed speculator has a positive probability to buy, then according to the Single Crossing Property, the uninformed and positively informed speculator will choose to buy for sure. Since $Q = -1$ will only appear when the speculator choose to sell or not trade, thus the firm and market maker will expect it to be a low type signal after this trading result and choose not to invest. Even though there is a case when all three types speculator choose to buy, which leaves $Q = -1$ to be an out-of-equilibrium trading result, among them the negatively informed speculator would have the largest incentive to deviate. D1 condition implies that when $Q = -1$ unexpected appears, firm and market maker will believe it to be conducted by the negatively informed speculator, and again choose not to invest.

As for the cases when the negatively informed speculator will never choose to buy, then the trading strategy of uninformed speculator will become more crucial. If the uninformed speculator has a positive probability to buy, then the Single Crossing Property tells us that the positively informed speculator will choose to buy for sure, which leaves $Q = -1$ to be an either low type signal or just no signal at all. Since by assumption, the firm will optimally reject the project after orders that do not distinguish between the negatively informed and uninformed speculator, thus the firm manager will still not invest when $Q = -1$ in this case.

So the only remained cases will be both the negatively informed and uninformed speculator never choose to buy. However, in these cases we also need the positively informed speculator's probability of buying to be low enough, which in other words is that the positively informed speculator's probability of selling and doing nothing to be sufficient high, to make the firm believe that investment is worthy when total order flow $Q = -1$.

In sum, the only cases for firm to invest when $Q = -1$, and the market maker to set a positive price for the stock is when the positively informed speculator has a low enough probability of buying. Thus by contrapositive, we will have the Lemma above.

Lemma 2.3. *In any perfect Bayesian equilibrium, if* $p_h^s < 1 - \frac{2\bar{V}}{\alpha V^+}$*, then* $E\left(V_\omega \,|Q = -2\right) < 0$*, the firm will not choose to invest when total order flow* $Q = -2$*, the market maker will set a price to equal zero,* $P_{-2} = 0$*.*

Proof. See Appendix. ☐

The intuition behind this Lemma is similar to what we discussed in Lemma 2.2. Once the positively informed speculator chooses not to sell, then the trading order $Q = -2$ can be only brought by either the negatively informed speculator or the uninformed speculator. This is because in order to have a $Q = -2$, we need both speculator and noise trader to submit a selling order, if the speculator chooses not to sell, then even when the noise trader sells, the total order flow will be at least $Q = -1$. So like we mentioned before, the firm will not invest when the trading quantity implies it to be either a negatively informed speculator or an uninformed speculator. And even though there would be a case when every types speculator choose not to sell, which leaves $Q = -2$ to be an out-of-equilibrium trading result, D1 implies that the firm and market maker will still treat $Q = -2$ as a low type signal and reject the project.

Next consider the cases when the positively informed speculator has a positive probability to sell, then by Single Crossing Property we have the uninformed and negatively informed speculator will choose to sell for sure. Thus in order to convey an investment signal when $Q = -2$, the probability of selling for the positively informed speculator should not only be positive, but also surpass a certain level. In other words, the only cases for firm to invest when $Q = -2$, and the market maker to set a positive price for the stock is when the positively informed speculator has a high enough probability of selling. Again by contrapositive, we will have the result as Lemma 2.3.

Lemma 2.4. *In any perfect Bayesian equilibrium, we will have* π_1=1 *and* $P_1 > 0$*, the firm will always invest when total order flow* $Q = 1$*, and the market maker will set the price greater than zero.*

Proof. See Appendix. □

The intuition behind Lemma 2.4 is this. As long as the positively informed speculator chooses not to sell, then whether the positively informed speculator decides to buy or do nothing, there will always be a probability of $\frac{1}{3}$ that the total order flow $Q = 1$. Since the average of the high type and low type investment value is still greater than zero, so the high type signal has a rather large incentive to encourage the firm to invest. That is to say, if the firm manager sees a trading quantity result which could always be potentially derived from the positively speculator's strategy, then the firm will choose to invest. Notice that $Q = 0$ in Lemma 2.1 is a special case that follows this property.

As for the cases when the positively informed speculator has a positive probability to sell, again by Single Crossing Property, we can see that the uninformed speculator and negatively informed speculator will both sell for sure, which leaves $Q = 1$ to be a trading result that could only be brought by the positively informed speculator. Thus the firm will optimally accept the project when $Q = 1$, and the market maker will set a positive price in these cases. Even the positively informed speculator chooses to sell for sure, which leaves

$Q = 1$ to be an out-of-equilibrium trading result, then D1 implies that firm and market maker will believe it to be conducted by the positively informed speculator, and still choose to invest.

In sum, once we see the total order flow $Q = 1$, the firm will then choose to invest in all possible cases. Thus we have the result in Lemma 2.4.

Lemma 2.5. *In any perfect Bayesian equilibrium, we will have* π_2=1 *and* $P_2 > 0$*, the firm will always invest when total order flow* $Q = 2$*, and the market maker will set the price greater than zero.*

Proof. See Appendix. □

The intuition behind this Lemma is similar to Lemma 2.4. Since $Q = 2$ could only appear when both speculator and noise trader submit a buying order, if the speculator chooses not to buy, then even when the noise trader buys, the total order flow will be at most $Q = 1$. From what we discussed before, we can easily find that as long as the firm manager believes a trading result could not be potentially brought by the negatively informed speculator, then the firm will choose to invest. So if the negatively informed speculator chooses not to buy, then the firm will invest when $Q = 2$ for sure. And if the negatively informed speculator chooses to buy, then by Single Crossing Property, we have the uninformed and positively informed speculator will both choose to buy for sure. Thus in these cases, the firm will already have a strong enough confidence to invest due to the behavior of positively informed and uninformed speculator.

In sum, $Q = 2$ is a must-invest signal for the firm, since the firm will choose to invest in all possible cases for $Q = 2$. Thus we will have the result in Lemma 2.5.

According to all the Lemma 2.1 to Lemma 2.5, we will now have a general understanding for the firm's investment strategies. And if we look closely at Lemma 2.2 and Lemma 2.3, we can see that, as long as the firm strictly choose not to invest when $Q = -1$, then it will also strictly choose not to invest when $Q = -2$ (not vice versa); and as long as the firm strictly choose to invest when $Q = -2$, then it will strictly choose to invest when $Q = -1$ (not vice versa). Here the "strictly" means the firm will not mix in that case, instead it will always choose a pure strategy. Combine with the Lemma 2.1, 2.4 and 2.5, we can see there would be a positive relationship between the trading quantity and what profitability the firm believes it will be. In other words, a larger quantity of total demand order will give the firm manager a higher signal of the profitability of the project., and thus if a specific trading quantity leads the firm to invest, then any number of trading quantity beyond will also lead the firm to invest. This result is rather intuitive in real market. A higher quantity of total order demand reflects that the market believes the firm will have a good prospect in the future, and pushes the stock price up consequently.

Now that we have analyzed the strategies of the firm, next we will focus on the speculator's side. We will first solve for the equilibrium when the initial short position is not revealed, and then compare it with the cases when the initial short position is revealed.

2.3 Benchmark Results When Initial Short Position of Speculator is Not Revealed

Here we consider the benchmark model: the case when the initial short position of speculator is not revealed. In this case, the firm and market maker have no information on the initial position of the speculator. So they will expect in an unbiased way that the speculator initially has zero position on the stock.

2.3.1 Equilibrium Analysis From the Firm's Point of View

Since in the firm and market maker's belief, the speculator has no initial position on the stock, so we will first begin with the discussion of the speculator's strategy for $s = 0$.

Claim 2.3. *In any perfect Bayesian equilibrium with the initial short position $s = 0$, we will have $\Delta_h^{b,n} > 0$ and $\Delta_h^{n,s} > 0$, the positively informed speculator will always choose to buy.*

Proof. See Appendix. □

Since there is no initial short position, then the positively informed speculator's payoff in this game will be just her trading profit. So if the positively informed speculator chooses to do nothing, she will get zero profit. As we discussed in previous section, the strategy of the positively informed speculator plays an important role on firm's investment decision, specifically speaking, if the positively informed speculator chooses to buy, then the firm will invest for $Q = 0, 1, 2$; if the positively informed speculator chooses to sell, then the firm will invest for $Q = -2, -1, 0$. However, selling will always gain non-positive trading profit for the positively informed speculator. This is because after the firm taking the investment, the project's value will then be realized to equal V^+ due to the high profitability. Since the market maker will never set a price higher than V^+, the positively informed speculator could not earn any positive profit from selling. Besides, since the price $P_0 = \bar{V}$ is strictly less than V^+, thus the positively informed speculator will actually suffer a loss when $Q = 0$. Therefore the expected payoff of selling for positively informed speculator will be strictly less than zero. Apply the same logic, we can see that the expected payoff of buying for positively informed speculator will be strictly higher than zero. Hence, in sum, the positively informed speculator will always choose to buy when there is no initial short position.

Claim 2.4. *In any perfect Bayesian equilibrium with the initial short position $s = 0$, we will have $\Delta_l^{b,n} < 0$ and $\Delta_l^{n,s} < 0$, the negatively informed speculator will always choose to sell.*

Proof. See Appendix. □

The intuition behind this Claim is similar to the logic of Claim 2.3. The negatively informed speculator knows that the profitability is low type for sure. So from her point of view, buying will always give her a negative trading profit. The reason is as follow. Since the firm will always choose to invest when $Q = 0, 1, 2$ (from Lemma 2.1, 2.4, 2.5), then after the investment taking place, the project's value will be realized to equal V^-. Notice that the market maker will never set a price below zero (this is because if the market maker sets a negative price, then the firm can always choose to reject the project to prevent the expected loss, which leaves a lower bound zero for the price), so the negatively informed speculator will always earn a negative profit from buying. Likewise, selling will always give the negatively informed speculator a non-negative trading profit. The reason why I use "non-negative" instead of "positive" is because, unlike the cases from buying, the firm may choose not to invest when the firm sees a lower total order flow, which then leaves a zero profit for the negatively informed speculator. However, since $Q = 0$ is also a potential trading result when the negatively informed speculator chooses to sell, then Lemma 1 ensures a positive profit for the negatively informed speculator in that case. So the expected payoff from selling will be strictly positive. Therefore, without any initial short position, the negatively informed speculator will choose to sell.

Claim 2.5. *In any perfect Bayesian equilibrium with the initial short position $s = 0$, we will have $\Delta_{\emptyset}^{b,n} < 0$ and $\Delta_{\emptyset}^{n,s} = 0$, the uninformed speculator will mix between doing nothing and selling.*

Proof. See Appendix. □

The intuition behind this Claim is based on the result in Claim 2.3 and 2.4. Now that the positively informed speculator chooses to buy, and the negatively informed speculator chooses to sell, then what the uninformed speculator chooses to do will make no difference in the firm's investment strategy. The firm will always invest when $Q = 0, 1, 2$ and not invest when $Q = -1, -2$. If that is the case, then the uninformed speculator will earn a negative profit from buying. The reason is the prices for $Q = 1, 2$ are too high. Since negatively informed speculator will always choose to sell, then when $Q = 1, 2$, the firm and market maker will treat it as either positively informed or uninformed. So the price P_1 and P_2 will be both greater than $\bar{V}$. However, the expected value will be only $\bar{V}$ from the uninformed speculator's assessment, thus she will suffer a loss if she chooses to buy and the resulting quantity appears to be 1 or 2. Even when $Q = 0$, the price equals $\bar{V}$ by Lemma 2.1, the uninformed speculator could not earn a positive profit from buying (she will actually earn a zero profit), so in any cases with $s = 0$, the uninformed speculator will never choose to buy.

If the uninformed speculator instead chooses to sell, then her payoff will be zero. This would be much easier to understand. Since the firm will not invest when $Q = -1, -2$, thus the price P_{-1} and P_{-2} will be zero, the uninformed speculator earns zero profit in those cases. As for the cases when $Q = 0$, the price $P_0 = \bar{V}$ matches the valuation of the uninformed speculator herself, which also makes selling unprofitable to the uninformed speculator. In sum, the uninformed speculator will earn zero expected payoff from selling, which is just the same as doing nothing (notice that the agents in this model are risk neutral), so the uninformed speculator will mix between selling and doing nothing.

As we mentioned before, when the initial short position of speculator is not revealed, firm and market maker will assume the speculator initially has no position on the stock. So without any further information, the firm and market maker's decisions will base on the result in Claim 2.3-2.5. Thus according to the Lemmas in previous section, we will draw the following Claim naturally.

Claim 2.6. *In any perfect Bayesian equilibrium, when initial short position is not revealed, the following holds*

Investment strategy: *Firm will invest only when total order flow $Q = 0, 1, 2$, and will not invest when $Q = -1, -2$.*

Pricing strategy: *Market maker will set price as*

$$P_{-2} = P_{-1} = 0$$

$$P_0 = \bar{V}$$

$$P_2 = V^+$$

$$P_1 = \frac{\frac{\alpha}{2}V^+ + p_\emptyset^n(1-\alpha)\bar{V}}{\frac{\alpha}{2} + p_\emptyset^n(1-\alpha)}$$

Proof. See Appendix. □

The firm's investment strategy is rather intuitive. In firm's belief, the financial markets will provide information that guides its real investment decisions. So a higher total order flow, $Q = 1, 2$, conveys a high type signal, which then gives the firm confidence to invest. Meanwhile a lower total order flow, $Q = -1, -2$, delivers a low type signal, which will suggest the firm to reject the project. And for the remained cases with $Q = 0$, even if it contains no information, since the ex-ante NPV of the project is positive, the firm will still choose to invest when $Q = 0$ appears.

The logic behind the pricing strategy of the market maker is basically the same. From Claim 2.3-2.5, the only buyer would be the positively informed speculator, thus the price P_2 should be V^+. Since the firm will reject the investment when $Q = -1, -2$, then the price P_{-1} and P_{-2} should be set to equal zero. And by Lemma 2.1, the price P_0 is always equal $\bar{V}$, so the only remained is the price when $Q = 1$. Notice that according to Claim 2.5, the uninformed speculator is mixing between selling and doing nothing. As the uninformed speculator's probability of buying increases, the trading quantity $Q = 1$ will become less informative, which will then squeeze down the price. On the contrary, as the uninformed speculator's probability of buying decreases, it will be more easily for the firm to distinguish between the positively informed speculator and the uninformed one, which thus pushes the price up. After some mathematical derivation, we will have the resulting summary above.

By now, we solved for the equilibrium strategies for the firm and market maker, next we will start with the analysis from the speculator's side.

2.3.2 Equilibrium Analysis From the Speculator's Point of View

Since the initial short position is not revealed, then there will be some information asymmetry between the speculator and the firm. Obviously, the speculator will have a better understanding for her own situation, and thus will take advantage of what the firm and market maker believe to maximize her own utility.

Claim 2.7. *In any perfect Bayesian equilibrium, when the initial short position* $s > 0$ *and is not revealed, we will have* $\Delta_l^{b,n} < 0$ *and* $\Delta_l^{n,s} < 0$*, the negatively informed speculator will always choose to sell.*

Proof. See Appendix. □

The intuition behind this Claim is straightforward. If the initial short position s is greater than zero, then the speculator will need to pay back money at the end of period 1 to close her short position. So the speculator now has an incentive to lower down the stock price in order to gain from her initial short position. As for the negatively informed speculator, from Claim 2.4, since she already prefers to sell when $s = 0$, thus the introduction of the short position will even give her a stronger motivation to sell. Moreover, the negatively informed speculator knows that the firm will not invest when $Q = -1, -2$ and the market maker will set both price to equal zero. So if the negatively informed speculator chooses to sell, she will earn both the highest trading profit (from selling) and the highest pay-back profit (from closing her short position). Therefore the negatively informed speculator will still choose to sell when $s > 0$ and is not revealed.

Claim 2.8. *In any perfect Bayesian equilibrium, when the initial short position* $s > 0$ *and is not revealed, we will have* $\Delta_\emptyset^{b,n} < 0$ *and* $\Delta_\emptyset^{n,s} < 0$*, the uninformed speculator will always choose to sell.*

Proof. See Appendix. □

The intuition behind this result is the same as before. The initial short position will encourage the uninformed speculator to attack the stock price by selling. So even the uninformed speculator is indifference between selling and doing nothing as we mentioned in Claim 2.5, an increase in her short position s will make selling more attractive. This is because although the uninformed speculator could only earn a zero trading profit from selling, which is the same as doing nothing, she can lower down the stock price through that way, and thus earn a higher pay-back profit. So when the initial short position $s > 0$ is not revealed, the uninformed speculator will choose to sell.

Claim 2.9. *In any perfect Bayesian equilibrium, when the initial short position $s > 0$ and is not revealed, for the positively informed speculator, the following holds*

Trading strategy: *If $s < 1-\frac{\bar{V}}{V^+}$, then the positively informed speculator will always choose to buy; if $s > 1-\frac{\bar{V}}{V^+}$, then the positively informed speculator will always choose to sell.*

Proof. See Appendix. □

The intuition behind this trading strategies is rather clear. The positively informed speculator has a buying incentive due to her high type signal. The stock price would be lower than the positively informed speculator's assessment, which will give her a positive trading profit from buying. However, the initial short position will have an opposite effect on the positively informed speculator's decision, i.e. the burden of closing the short position will lead her into selling. And as the short position goes up, the pressure from the selling side will continually become stronger. So when the initial short position s goes beyond some threshold (it would be $1-\frac{\bar{V}}{V^+}$ in this case), the positively informed speculator will eventually deviate to selling. If the short position happens to be equal $1-\frac{\bar{V}}{V^+}$, then the positively informed speculator will actually mix between buying, selling and doing nothing. But with prices and initial short position all given constant, it seems to be non-generic in this case, so we omit it in the Claim 2.9.

As we can see, the financial market will lose some efficiency if the short position is not revealed, especially when the short position is relatively large. The trading result will become more misleading as the speculator's initial short position goes up. Since with a large initial short position, even the positively informed speculator will choose to sell, thus the firm may mistakenly reject some projects which are potentially worth investing.

So in the next section, we will discuss the equilibrium when the initial short position is revealed, and then compare these two results in detail.

2.4 Results When Initial Short Position of Speculator is Revealed

We have already shown in the last section that when the initial short position of speculator is not revealed, the allocation results induced by prices is not efficient. Here we will examine what will happen if government requires speculator to reveal her short position s.

Claim 2.10. *In any perfect Bayesian equilibrium, when the initial short position $s > 0$ is revealed, we will have $\Delta_l^{b,n} < 0$ and $\Delta_l^{n,s} < 0$, the negatively informed speculator will always choose to sell.*

Proof. See Appendix. □

The intuition behind Claim 2.10 is as flow. Since the initial short position is now revealed, the speculator needs to reconsider the firm and the market maker's strategies. The difference is that now the firm may choose to invest even when $Q = -1, -2$. However, even if the firm chooses to invest for Q equals either -1 or -2 (this could happen when the positively informed speculator has a high enough probability of selling), the negatively informed speculator's trading profit from selling will rather increase. Notice that the investment of the firm will actually push the stock price up, and since the negatively informed speculator knows the profitability would be low type for sure, so a positive price P_{-1} or P_{-2} will ensure the trading profit of selling to be greater than zero. As doing nothing yields zero trading profit, and buying will in the contrary always earn a negative trading profit, thus in this respect, the negatively informed speculator has an incentive to sell. Also if the firm chooses not to invest for $Q = -1, -2$, then the firm's strategy would be the same as in the last section (Lemma 2.1, 2.4, 2.5 tell us the firm will always invest when $Q = 0, 1, 2$), so from the Claim 2.7 we will have selling is still optimal for the negatively informed speculator.

Next consider the negatively informed speculator's strategies from paying-back side. Since the speculator needs to close her initial short position at the end of period 1, so a lower price would be better for her to save the pay-back money. If the positively informed speculator has a positive probability to sell, then according to the Single Crossing Property, the uninformed and negatively informed speculator will sell for sure. Since the positively informed speculator is the only type who will choose to buy, the price P_1 and P_2 will be set to equal V^+. Even when the positively informed speculator also sells for sure, which leaves $Q = 1, 2$ to be out-of-equilibrium trading results, D1 implies that the firm and market maker will believe it to be conducted by the positively informed speculator, and the price should be still equal V^+. So if the positively informed speculator has a positive probability to sell, the price will be strictly lower when $Q = -1, -2$, therefore the initial short position will encourage the negatively informed speculator to sell.

If the positively informed speculator never chooses to sell, then when $Q = -2$, the firm will expect it to be either uninformed or negatively informed, and thus reject the project. The market maker will then set price P_{-2} to equal zero. Since the initial short position inspire the negatively informed speculator to squeeze down the stock price, then selling would be optimal to close her short position in this case.

In sum, the negatively informed speculator has incentives to sell from both trading profit side and paying-back money side. Thus when the initial short position $s > 0$ is revealed, the negatively informed speculator will always choose to sell.

Claim 2.11. *In any perfect Bayesian equilibrium, when the initial short position $s > 0$ is revealed, we will have $\Delta_{\emptyset}^{b,n} < 0$, the uninformed speculator will never choose to buy.*

Proof. See Appendix. □

Claim 2.12. *In any perfect Bayesian equilibrium, when the initial short position $s > 0$ is revealed, we will have $p_{\emptyset}^{s} = 1$, the uninformed speculator will always choose to sell.*

Proof. See Appendix. □

The reason why I divided the strategies of the uninformed speculator into two Claims is that the result in Claim 2.12 is actually based on what we proved in Claim 2.11. The intuition for the uninformed speculator not to buy is straightforward. Since we already proved in the last section that buying is a strongly dominated strategy when $s = 0$, thus an introduction of the initial short position will even further make the uninformed speculator not to buy.

The reason why the uninformed speculator will switch from mixing to selling is a little bit tricky. Since the negatively informed speculator will always choose to sell, the price for $Q = -1, -2$ will not go beyond the uninformed speculator's valuation (which is $\bar{V}$). So from the trading profit side, doing nothing is always at least as good as selling. However, compare selling with doing nothing from the paying-back money side, the price P_1 will be strictly larger than the price P_{-2} (the firm will expect it to be either positively informed or uninformed when $Q = 1$), which makes selling more attractive to the uninformed speculator. Thus there would be trade-off between these to strategies. But notice that the uninformed speculator will not have more information than what the firm and the market maker know, so the profit she can gain from trading is to some extent limited. Thus the pressure from the paying-back money side would be stronger, which finally leads the uninformed speculator to sell. A more rigorous mathematical proof is included in the Appendix.

Now that we have already figured out what the negatively informed speculator and the uninformed speculator will choose to do when the initial short position $s > 0$ is revealed, next we will discuss the strategies of the positively informed speculator. As for the positively informed speculator, the trade-off effect we mentioned above will be even more complex. So we need to solve for all possible cases in order to analysis the behavior of the positively informed speculator.

Claim 2.13. *In any perfect Bayesian equilibrium, when the initial short position $s > 0$ is revealed, the following holds for the positively informed speculator.*

Trading strategy.

- *If* $s < 1 - \frac{\bar{V}}{V+}$, *then* $p_h^b = 1$, *the positively informed speculator will always choose to buy.*
- *If* $1 - \frac{\bar{V}}{V+} < s \leq 1$, *then* $p_h^b + p_h^n = 1$, *the positively informed speculator will mix between buying and doing nothing.*
- *If* $1 < s < 2 - \frac{2\bar{V}}{V+}$, *then* $p_h^n = 1$, *the positively informed speculator will always choose not to trade.*
- *If* $2 - \frac{2\bar{V}}{V+} \leq s \leq 3$, *then* $p_h^n + p_h^s = 1$, *the positively informed speculator will mix between doing nothing and selling.*
- *If* $3 < s$, *then* $p_h^s = 1$, *the positively informed speculator will always choose to sell.*

Proof. See Appendix. □

This Claim is a summary of the positively informed speculator's strategies in all different cases. Since the initial short position is now revealed, the positively informed speculator could not pretend to be a negatively informed speculator without any cost. The firm and the market maker have already realized that even the positively informed speculator will have an incentive to sell when the initial short position is large enough. So the positively informed speculator must take the trade-off between the trading profit loss and the paying-back money gain into consideration.

The general idea between the Claim 2.13 is that as s goes up, the positively informed speculator will put more weight on the gain of closing her short position, and thus gradually move from a higher action (buying) towards a lower action (selling). To be more specific, starting with a low volume of short position, the positively informed speculator will still choose to buy. After the short position s goes beyond $1 - \frac{\bar{V}}{V+}$, the positively informed speculator will begin to mix between buying and doing nothing. If the short position s increases further above 1, then doing nothing will be optimal for the positively informed speculator. And once the short position s reaches $2-\frac{2\bar{V}}{V^+}$, the positively informed speculator will start to mix between selling and doing nothing. Until finally when the short position s becomes larger than 3, the positively informed speculator will always choose to sell (notice all types speculators will sell at this time).

After solving for all the strategies of the different types speculator, we could then turn our attention to the firm and market maker's strategies. Since the Lemma 2.1-2.5 gives us a general guidance of what the firm and market maker will do, so next we only need to apply these Lemmas to draw some conclusions.

Claim 2.14. *In any perfect Bayesian equilibrium, when the initial short position $s > 0$ is revealed, the following holds for the firm and the market maker.*

Investment strategy.

- *If $s < 1 - \frac{\bar{V}}{V^+}$, then firm will invest when $Q = 0, 1, 2$, and will not invest when $Q = -1, -2$.*
- *If $1 - \frac{\bar{V}}{V^+} < s < 2 - \frac{2\bar{V}}{V^+}$, then firm will invest when $Q = -1, 0, 1, 2$, and will not invest when $Q = -2$.*
- *If $2 - \frac{2\bar{V}}{V^+} \leq s$, then firm will always choose to invest, i.e. invest when $Q = -2, -1, 0, 1, 2$.*

Pricing strategy.

- *If $s < 1 - \frac{\bar{V}}{V^+}$, then market maker will set the prices as*

$$P_1 = P_2 = V^+$$

$$P_0 = \bar{V}$$

$$P_{-1} = P_{-2} = 0$$

- *If $s < 1 - \frac{\bar{V}}{V^+}$, then market maker will set the prices as*

$$P_1 = P_2 = V^+$$

$$P_0 = \bar{V}$$

$$P_{-1} = \frac{\left[\bar{V} - p_h^b \frac{\alpha}{2} V^+\right]}{\left[1 - p_h^b \frac{\alpha}{2}\right]}$$

$$P_{-2} = 0$$

- *If* $s < 1 - \frac{\bar{V}}{V^+}$*, then market maker will set the prices as*

$$P_1 = P_2 = V^+$$

$$P_0 = P_{-1} = \bar{V}$$

$$P_{-2} = \frac{\left[\bar{V} - (1 - p_h^s) \frac{\alpha}{2} V^+\right]}{\left[1 - \left(1 - p_h^s\right) \frac{\alpha}{2}\right]}$$

Proof. See Appendix. □

The intuition behind this conclusion is straightforward. Since now the firm knows that both the negatively informed and the uninformed speculators will choose to sell, the firm manager will pay her attention only on the positively informed speculator's behavior.

When the short position s is small, the financial market could still thought to be efficient. The firm will follow the guidance of the total order flow, choose to invest when $Q = 0, 1, 2$ and reject the project when $Q = -1, -2$.

If the short position S becomes large enough to make the positively informed speculator mix between buying and doing nothing. then the firm might also put $Q = -1$ into investment grade. According to Lemma 2.1, 2.4, 2.5, the firm will always invest when $Q = 0, 1, 2$, so the firm will only reject the project when $Q = -2$.

Finally if the short position becomes so large that even the positively informed speculator will always choose to sell, then the firm will invest for all possible quantities. This is because the financial market conveys no information at this stage, the firm manager will make decision based only on her own judgment (which always leads to an investing due to the positive ex-ante NPV).

The intuition behind the market maker's pricing strategies is almost the same as the firm. The market maker will always set the price to equal zero if the firm chooses not to invest. And since the positively informed speculator is the only type who will choose to buy, so the market maker will set price $P_1 = P_2 = V^+$. Also from the Lemma 1, we have $P_0 = \bar{V}$. For the rest undetermined price, the market maker will set it to equal the expected value of V_ω. All summary results are presented in the Claim above.

After the reveal of the short position, comparing with the results in the last section, we can see the situation has been truly improved. The firm now will not reject the projects just because a potentially manipulated low price. Even when the initial short position goes beyond some extremely high level, which prevent the firm from receiving any information from the financial market, the firm would still rely on its own ex-ante judgment to make the investment decisions. Figure 2.4 contains detailed comparison of the results when s is revealed and not revealed.

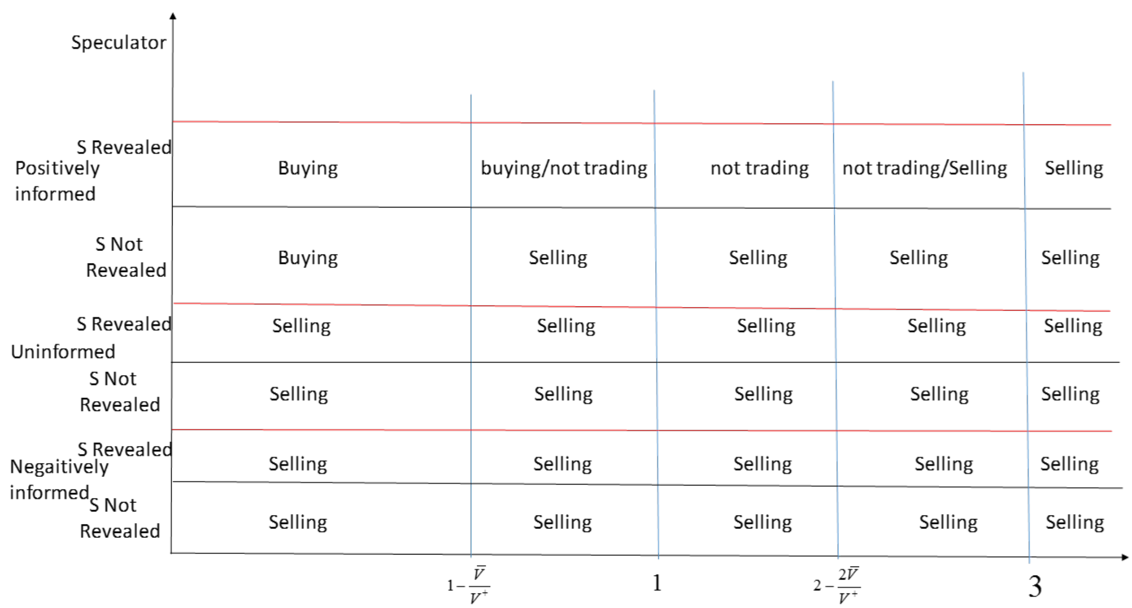

Figure 2.4: Comparison of the Results When Initial Short Position is Revealed and Not Revealed

2.5 Conclusion

It is commonly believed that financial markets provide information that guides real investment decisions. However, in this paper we show that the presence of the initial short position may cause the price to fail in fulfilling this function. By comparing the two equilibrium results before and after the reveal of the short position, we provide some regulation advices in the financial market.

Specifically, we study the behavior of a speculator with an initial short position, who may or may not be informed about the profitability of the project. When the initial short position is not revealed, the negatively informed speculator and the uninformed speculator will choose to sell, the positively informed speculator may also choose to sell if the short position is relatively large. This is because the introduction of the initial short position will give the speculator an incentive to manipulate the financial market. In other words, the uninformed speculator and the positively informed speculator may pretend to be negatively informed. Through this way, she can squeezed down the stock price and thus reduce the money she needs to pay back for closing her short position at the end of period 1. However, since the firm has no idea about the initial position of the speculator, this information asymmetry will cause the pricing mechanism to be less efficient. The firm expects the positively informed speculator to buy and the negatively informed speculator to sell, which then let the total order flow convey a signal about the profitability of the project. Now that the initial position is not revealed, the trading quantity will become misleading to the firm manager, prevent her from making the right investment decisions. In other words, when the initial short position s is large enough, all three types speculators will choose to sell, thus the total order flow Q could only be either -2, -1, or 0. Notice that the firm will not choose to invest when $Q = -1, -2$, so the firm may reject some projects which are essentially worth investing.

After the reveal of the initial short position, we can see that the information asymmetry has been eliminated, so the firm may also choose to invest when the trading quantity is low. The market efficiency has been improved from two aspects. First, the behavior of the positively informed speculator will be less aggressive when the initial short position is large. Since the positively informed speculator could not pretend to be negatively informed without any cost, so the trade-off effect between trading profit and paying-back money will make the positively informed speculator move slower and more smooth from a higher action (buying) towards a lower action (selling). Second, once the initial short position is revealed, the firm can make better use of the financial market's information. A lower trading quantity may not entirely represent a bad signal, it might also conducted by the needs of closing initial short position from the positively informed speculator. Even if when the initial short position is very large, which makes all types speculators sell for sure, the firm manager will put her own judgment into consideration and try not to reject the project easily like before.

In sum, the main result of this paper is that the short position in the financial market should be revealed publicly. Eliminating the information asymmetry will help improve the efficiency of the financial market. Also notice that when the short position is extremely large, the efficiency of the financial market will be restricted even with the revealing regulation. So further research is needed to answer the question of whether the maximum amount of the short position for a single trader should be limited or not.

2.6 Appendix: Proofs of the Lemmas and Claims

Let

$$\mu_\omega^q = \Pr\left(\omega \left| Q = q \right.\right) = \frac{\Pr\left(Q = q \left| \omega \right.\right) \Pr\left(\omega\right)}{\Pr\left(Q = q\right)}$$

denote the probability of the signal given the total order flow.

The unconditional probability of the signal is given by

$$\Pr\left(\omega\right) = \begin{cases} \frac{\alpha}{2} \quad if \quad \omega \in \{h, l\} \\ \\ 1 - \alpha \quad if \quad \omega = \emptyset \end{cases}$$

And the speculator's strategy will be listed as

$$\Pr\left(D_S = 1 \,|\omega\right) = p_\omega^b$$

$$\Pr\left(D_S = 0 \,|\omega\right) = p_\omega^n$$

$$\Pr\left(D_S = -1 \,|\omega\right) = p_\omega^s$$

Thus we will have a summary table of the speculator's strategy probability given the signal and the total order flow in Table A2.6.1.

Table A2.6.1: Probability of the Speculator's Strategy Given Signal and Total Order Flow

Total order flow	Sell	Not trade	Buy
-2	p_ω^s	0	0
-1	p_ω^n	p_ω^s	0
0	p_ω^b	p_ω^n	p_ω^s
1	0	p_ω^b	p_ω^n
2	0	0	p_ω^b

Since $\Pr\left(Q = q \,|\omega\right)$ is computed as

$$\Pr\left(Q = q \,|\omega\right) = \frac{1}{3}\sum_{d=-1}^{1} \Pr\left(D_S = q - d \,|\omega\right)$$

then we will also have a table of probability of the total order flow given signal in Table A2.6.2

Proof of Lemma 2.1

Proof. When $Q = 0$, the expected value of V_ω is

$$\begin{aligned} E\left(V_\omega \,|Q = 0\right) &= \mu_h^0 V^+ + \mu_l^0 V^- + \mu_\emptyset^0 \bar{V} \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q = 0\right)}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \end{aligned}$$

Table A2.6.2: Probability of the Realized Total Order Flow Given Different Signal

Total order flow	High signal	No signal	Low signal
-2	$\frac{1}{3}p_h^s$	$\frac{1}{3}p_\emptyset^s$	$\frac{1}{3}p_l^s$
-1	$\frac{1}{3}\left(p_h^s + p_h^n\right)$	$\frac{1}{3}\left(p_\emptyset^s + p_\emptyset^n\right)$	$\frac{1}{3}\left(p_l^s + p_l^n\right)$
0	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
1	$\frac{1}{3}\left(p_h^b + p_h^n\right)$	$\frac{1}{3}\left(p_\emptyset^b + p_\emptyset^n\right)$	$\frac{1}{3}\left(p_l^b + p_l^n\right)$
2	$\frac{1}{3}p_h^b$	$\frac{1}{3}p_\emptyset^b$	$\frac{1}{3}p_l^b$

Since the probability of $Q = 0$ can be computed as

$$\Pr\left(Q=0\right) = \frac{1}{3}\left[\left(1-\alpha\right)\left(p_\emptyset^b + p_\emptyset^n + p_\emptyset^s\right) + \frac{\alpha}{2}\left(p_h^b + p_h^n + p_h^s\right) + \frac{\alpha}{2}\left(p_l^b + p_l^n + p_l^s\right)\right] = \frac{1}{3}$$

Then we will have

$$E\left(V_\omega \left| Q=0\right.\right) = \frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V} = \bar{V} > 0$$

Thus the firm will invest for sure when total order flow $Q = 0$, and the market maker will set the price equal to expected asset value, which is $E\left(V_\omega \left| Q=0\right.\right) = \bar{V}$ □

Proof of Lemma 2.2

Proof. Since $E\left(V_\omega \left| Q=q\right.\right) = \mu_l^q V^- + \mu_h^q V^+ + \mu_\emptyset^q \bar{V}$, then from the table above we will have

$$\begin{aligned} E\left(V_\omega \left| Q=-1\right.\right) &= \mu_h^{-1}V^+ + \mu_l^{-1}V^- + \mu_\emptyset^{-1}\bar{V} \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\frac{\alpha}{2}V^+\left(1-p_h^b\right) + \frac{\alpha}{2}V^-\left(1-p_l^b\right) + \left(1-\alpha\right)\bar{V}\left(1-p_\emptyset^b\right)\right] \end{aligned}$$

Case 1. If $0 < p_l^b < 1$, then by SCP we will have $p_\emptyset^b = p_h^b = 1$.

The probability of $Q=-1$ will be

$$\begin{aligned}\Pr(Q=-1) &= \frac{1}{3}\left[(1-\alpha)\left(p_{\emptyset}^{n}+p_{\emptyset}^{s}\right)+\frac{\alpha}{2}\left(p_{h}^{n}+p_{h}^{s}\right)+\frac{\alpha}{2}\left(p_{l}^{n}+p_{l}^{s}\right)\right]\\ &= \frac{1}{3}\left[(1-\alpha)\left(1-p_{\emptyset}^{b}\right)+\frac{\alpha}{2}\left(1-p_{h}^{b}\right)+\frac{\alpha}{2}\left(1-p_{l}^{b}\right)\right]\\ &= \frac{1}{3}\left[\frac{\alpha}{2}\left(1-p_{l}^{b}\right)\right]\end{aligned}$$

So the expected value of V_ω given $Q=-1$ will be

$$\begin{aligned}E\left(V_\omega\,|Q=-1\right) &= \frac{1}{3}\frac{1}{\Pr(Q=-1)}\left[\frac{\alpha}{2}V^{+}\left(1-p_{h}^{b}\right)+\frac{\alpha}{2}V^{-}\left(1-p_{l}^{b}\right)+(1-\alpha)\bar{V}\left(1-p_{\emptyset}^{b}\right)\right]\\ &= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}\left(1-p_{l}^{b}\right)\right]}\left[\frac{\alpha}{2}V^{-}\left(1-p_{l}^{b}\right)\right]=V^{-}<0\end{aligned}$$

Case 2. If $p_{l}^{b}=1$, then by SCP we will have $p_{\emptyset}^{b}=p_{h}^{b}=1$. In this case all three types speculators will choose to buy, which leaves $Q=-1$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is negatively informed, thus the expected value of V_ω will be $E\left(V_\omega\,|Q=-1\right)=V^{-}<0$.

Case 3. If $p_{l}^{b}=0$ and $p_{\emptyset}^{b}>0$, then by SCP we will have $p_{h}^{b}=1$.

So the expected value of V_ω given $Q=-1$ will be

$$\begin{aligned}E\left(V_\omega\,|Q=-1\right) &= \frac{1}{3}\frac{1}{\Pr(Q=-1)}\left[\frac{\alpha}{2}V^{+}\left(1-p_{h}^{b}\right)+\frac{\alpha}{2}V^{-}\left(1-p_{l}^{b}\right)+(1-\alpha)\bar{V}\left(1-p_{\emptyset}^{b}\right)\right]\\ &= \frac{1}{3}\frac{1}{\Pr(Q=-1)}\left[\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\left(1-p_{\emptyset}^{b}\right)\right]\\ &\leq \frac{1}{3}\frac{1}{\Pr(Q=-1)}\left[\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\right]\end{aligned}$$

Since $\Pr(Q=-1)>0$, and by assumption $\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}<0$, then we will have $E\left(V_\omega\,|Q=-1\right)<0$.

Case 4. If $p_{l}^{b}=0$ and $p_{\emptyset}^{b}=0$, then the expected value of V_ω given $Q=-1$ will be

$$\begin{aligned}E\left(V_\omega\,|Q=-1\right) &= \frac{1}{3}\frac{1}{\Pr(Q=-1)}\left[\frac{\alpha}{2}V^{+}\left(1-p_{h}^{b}\right)+\frac{\alpha}{2}V^{-}\left(1-p_{l}^{b}\right)+(1-\alpha)\bar{V}\left(1-p_{\emptyset}^{b}\right)\right]\\ &= \frac{1}{3}\frac{1}{\Pr(Q=-1)}\left[\frac{\alpha}{2}V^{+}\left(1-p_{h}^{b}\right)+\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\right]\end{aligned}$$

Since $\Pr(Q=-1)>0$, then let $E\left(V_\omega\,|Q=-1\right)\geq 0$ will give us

$$\frac{\alpha}{2}V^{+}\left(1-p_{h}^{b}\right)+\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\geq 0$$

$$\Rightarrow p_h^b \leq \frac{2\bar{V}}{\alpha V^+}$$

In sum, in the first three cases, we all have $E(V_\omega|Q=-1) < 0$. And in the case 4, if we let $E(V_\omega|Q=-1) \geq 0$, then we will have $p_h^b \leq \frac{2\bar{V}}{\alpha V^+}$. Thus the expected value of V_ω given $Q=-1$ is greater than or equal to zero will imply that the probability of buying for positively informed speculator is less than or equal to $\frac{2\bar{V}}{\alpha V^+}$. That is

$$E(V_\omega|Q=-1) \geq 0 \Rightarrow p_h^b \leq \frac{2\bar{V}}{\alpha V^+}$$

From the contrapositive, we will have

$$p_h^b > \frac{2\bar{V}}{\alpha V^+} \Rightarrow E(V_\omega|Q=-1) < 0$$

If $E(V_\omega|Q=-1) < 0$, then firm will not invest, thus the market maker will set a price to equal zero. So we have

$$E(V_\omega|Q=-1) < 0 \Rightarrow P_{-1} = 0$$

□

Proof of Lemma 2.3

Proof. Since $E(V_\omega|Q=q) = \mu_l^q V^- + \mu_h^q V^+ + \mu_\emptyset^q \bar{V}$, then from the table above we will have

$$\begin{aligned} E(V_\omega|Q=-2) &= \mu_h^{-2} V^+ + \mu_l^{-2} V^- + \mu_\emptyset^{-2} \bar{V} \\ &= \frac{1}{3}\frac{1}{\Pr(Q=-2)}\left[\frac{\alpha}{2}V^+ p_h^s + \frac{\alpha}{2}V^- p_l^s + (1-\alpha)\bar{V} p_\emptyset^s\right] \end{aligned}$$

Case 1. If $p_h^s > 0$, then by SCP we will have $p_\emptyset^s = p_l^s = 1$.

The expected value of V_ω given $Q=-2$ will be

$$\begin{aligned} E(V_\omega|Q=-2) &= \frac{1}{3}\frac{1}{\Pr(Q=-2)}\left[\frac{\alpha}{2}V^+ p_h^s + \frac{\alpha}{2}V^- p_l^s + (1-\alpha)\bar{V} p_\emptyset^s\right] \\ &= \frac{1}{3}\frac{1}{\Pr(Q=-2)}\left[\frac{\alpha}{2}V^+ p_h^s + \frac{\alpha}{2}V^- + (1-\alpha)\bar{V}\right] \end{aligned}$$

Since $\Pr(Q=-2) > 0$, then let $E(V_\omega|Q=-2) \geq 0$ will give us

$$\frac{\alpha}{2}V^+ p_h^s + \frac{\alpha}{2}V^- + (1-\alpha)\bar{V} \geq 0$$

$$\Rightarrow p_h^S \geq 1 - \frac{2\bar{V}}{\alpha V^+}$$

Case 2. If $p_h^s = 0$ and $p_\emptyset^s > 0$, then by SCP we will have $p_l^s = 1$.

So the expected value of V_ω given $Q = -2$ will be

$$\begin{aligned} E\left(V_\omega \left| Q = -2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[\frac{\alpha}{2}V^+ p_h^s + \frac{\alpha}{2}V^- p_l^s + \left(1-\alpha\right)\bar{V}p_\emptyset^s\right] \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[\frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}p_\emptyset^s\right] \\ &\leq \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[\frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \end{aligned}$$

Since $\Pr\left(Q=-2\right) > 0$, and by assumption $\frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V} < 0$, then we will have $E\left(V_\omega \left| Q = -2\right.\right) < 0$.

Case 3. If $p_h^s = 0$, $p_\emptyset^s = 0$, and $p_l^s > 0$, then the probability of $Q = -2$ will be

$$\begin{aligned} \Pr\left(Q=-2\right) &= \frac{1}{3}\left[\left(1-\alpha\right)p_\emptyset^s + \frac{\alpha}{2}p_h^s + \frac{\alpha}{2}p_l^s\right] \\ &= \frac{1}{3}\left[\frac{\alpha}{2}p_l^s\right] \end{aligned}$$

So the expected value of V_ω given $Q = -2$ will be

$$\begin{aligned} E\left(V_\omega \left| Q = -2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[\frac{\alpha}{2}V^+ p_h^s + \frac{\alpha}{2}V^- p_l^s + \left(1-\alpha\right)\bar{V}p_\emptyset^s\right] \\ &= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}p_l^s\right]}\left[\frac{\alpha}{2}V^- p_l^s\right] = V^- < 0 \end{aligned}$$

Case 4. If $p_h^s = 0$, $p_\emptyset^s = 0$, and $p_l^s = 0$ then in this case all three types speculators will choose not to sell, which leaves $Q = -2$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is negatively informed, thus the expected value of V_ω will be $E\left(V_\omega \left| Q = -2\right.\right) = V^- < 0$.

In sum, in the case 2, 3, and 4 we all have $E\left(V_\omega \left| Q = -2\right.\right) < 0$. And in the case 1, if we let $E\left(V_\omega \left| Q = -2\right.\right) \geq 0$, then we will have $p_h^s \geq 1 - \frac{2\bar{V}}{\alpha V^+}$. Thus the expected value of V_ω given $Q = -2$ is greater than or equal to zero will imply that the probability of selling for positively informed speculator is greater than or equal to $1 - \frac{2\bar{V}}{\alpha V^+}$. That is

$$E\left(V_\omega \left| Q = -2\right.\right) \geq 0 \Rightarrow p_h^s \geq 1 - \frac{2\bar{V}}{\alpha V^+}$$

From the contrapositive, we will have

$$p_h^s < 1 - \frac{2\bar{V}}{\alpha V^+} \Rightarrow E\left(V_\omega \,|Q = -2\right) < 0$$

If $E\left(V_\omega \,|Q = -2\right) < 0$, then firm will not invest, thus the market maker will still set a price to equal zero. So we have

$$E\left(V_\omega \,|Q = -2\right) < 0 \Rightarrow P_{-2} = 0$$

□

Proof of Lemma 2.4

Proof. When $Q = 1$, the expected value of V_ω is

$$\begin{aligned} E\left(V_\omega \,|Q = 1\right) &= \mu_h^1 V^+ + \mu_l^1 V^- + \mu_\emptyset^1 \bar{V} \\ &= \frac{1}{3} \frac{1}{\Pr\left(Q = 1\right)} \left[\left(1 - p_h^s\right) \frac{\alpha}{2} V^+ + \left(1 - p_l^s\right) \frac{\alpha}{2} V^- + \left(1 - p_\emptyset^s\right)\left(1 - \alpha\right) \bar{V} \right] \end{aligned}$$

Case 1. If $p_h^s = 0$, then we will have

$$\begin{aligned} E\left(V_\omega \,|Q = 1\right) &= \frac{1}{3} \frac{1}{\Pr\left(Q = 1\right)} \left[\frac{\alpha}{2} V^+ + \left(1 - p_l^s\right) \frac{\alpha}{2} V^- + \left(1 - p_\emptyset^s\right)\left(1 - \alpha\right) \bar{V} \right] \\ &\geq \frac{1}{3} \frac{1}{\Pr\left(Q = 1\right)} \left[\frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- \right] = \frac{\alpha \bar{V}}{3 \Pr\left(Q = 1\right)} > 0 \end{aligned}$$

Case 2. If $0 < p_h^s < 1$, then by SCP we have $p_\emptyset^s = p_l^s = 1$

$$E\left(V_\omega \,|Q = 1\right) = \frac{1}{3} \frac{1}{\Pr\left(Q = 1\right)} \left[\left(1 - p_h^s\right) \frac{\alpha}{2} V^+ \right] > 0$$

Case 3. If $p_h^s = 1$, then by SCP we have $p_\emptyset^s = p_l^s = 1$. In this case all three types speculators will choose to sell, which leaves $Q = 1$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is positively informed, thus the expected value of V_ω will be $E\left(V_\omega \,|Q = 1\right) = V^+ > 0$.

In sum, $E\left(V_\omega \,|Q = 1\right)$ will be greater than zero in all three cases, so the firm will always invest when $Q = 1$ and the market maker will then set the price $P_1 > 0$. □

Proof of Lemma 2.5

Proof. When $Q = 2$, the expected value of V_ω is

$$\begin{aligned} E\left(V_\omega \,|Q = 2\right) &= \mu_h^2 V^+ + \mu_l^2 V^- + \mu_\emptyset^2 \bar{V} \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_h^b \frac{\alpha}{2} V^+ + p_l^b \frac{\alpha}{2} V^- + p_\emptyset^b \left(1-\alpha\right)\bar{V}\right] \end{aligned}$$

Case 1. If $p_l^b > 0$, then by SCP we have $p_\emptyset^b = p_h^b = 1$

$$\begin{aligned} E\left(V_\omega \,|Q = 2\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[\frac{\alpha}{2} V^+ + p_l^b \frac{\alpha}{2} V^- + \left(1-\alpha\right)\bar{V}\right] \\ &\geq \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[\frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- + \left(1-\alpha\right)\bar{V}\right] = \frac{\bar{V}}{3\Pr\left(Q=2\right)} > 0 \end{aligned}$$

Case 2. If $p_l^b = 0$, $p_\emptyset^b + p_h^b > 0$, then

$$E\left(V_\omega \,|Q = 2\right) = \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_h^b \frac{\alpha}{2} V^+ + p_\emptyset^b \left(1-\alpha\right)\bar{V}\right] > 0$$

Case 3. If $p_l^b = 0$, $p_h^b = 0$, and $p_\emptyset^b = 0$, then in this case all three types speculators will not choose to buy, which leaves $Q = 2$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is positively informed, thus the expected value of V_ω will be $E\left(V_\omega \,|Q = 2\right) = V^+ > 0$.

In sum, $E\left(V_\omega \,|Q = 2\right)$ will be greater than zero in all three cases, so the firm will always invest when $Q = 2$ and the market maker will then set the price $P_2 > 0$. □

Proof of Claim 2.3

Proof. If the initial short position $s = 0$, then the positively informed speculator's payoff of doing nothing will be zero, i.e. $U_h^n = 0$. So the relative payoff from buying vs. doing nothing is

$$\begin{aligned} \Delta_h^{b,n} &= U_h^b - U_h^n = V_h E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] \\ &= V_h \sum_{d=-1}^{1} \frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3} \\ &= \frac{1}{3}\sum_{d=-1}^{1}\left(V_h \pi_{d+1} - P_{d+1}\right) \end{aligned}$$

Notice that the market maker will set the price equal to expected asset value, so the price given total order flow $Q = q$ will be

$$P_q = \pi_q E\left(V_\omega \left| Q = q\right.\right)$$

Put it into $\Delta_h^{b,n}$ gives us

$$\Delta_h^{b,n} = \frac{1}{3} \sum_{d=-1}^{1} \pi_{d+1} \left(V_h - E\left(V_\omega \left| Q = d+1\right.\right)\right)$$

Since $E\left(V_\omega \left| Q = d+1\right.\right) \leq V_h$ for all d, and when $d = -1$, from Lemma 2.1 we have $E\left(V_\omega \left| Q = 0\right.\right) = \bar{V} < V_h$

$$\Rightarrow \Delta_h^{b,n} > 0$$

Also the relative payoff from doing nothing vs. selling is

$$\begin{aligned}
\Delta_h^{n,s} = U_h^n - U_h^s &= V_h E_D\left[\pi_{D-1}\right] - E_D\left[P_{D-1}\right] \\
&= V_h \sum_{d=-1}^{1} \frac{\pi_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_{d-1}}{3} \\
&= \frac{1}{3} \sum_{d=-1}^{1} \left(V_h \pi_{d-1} - P_{d-1}\right) \\
&= \frac{1}{3} \sum_{d=-1}^{1} \pi_{d-1} \left(V_h - E\left(V_\omega \left| Q = d-1\right.\right)\right)
\end{aligned}$$

Since $E\left(V_\omega \left| Q = d-1\right.\right) \leq V_h$ for all d, and when $d = 1$, from Lemma 2.1 we have $E\left(V_\omega \left| Q = 0\right.\right) = \bar{V} < V_h$

$$\Rightarrow \Delta_h^{n,s} > 0$$

In sum, both $\Delta_h^{b,n}$ and $\Delta_h^{n,s}$ are greater than zero, which implies that $\Delta_h^{b,s} = \Delta_h^{b,n} + \Delta_h^{n,s}$ is also greater than zero, so the positively informed speculator will always choose to buy. □

Proof of Claim 2.4

Proof. If the initial short position $s = 0$, then the negatively informed speculator's payoff of doing nothing will be zero, i.e. $U_l^n = 0$. So the relative payoff from buying vs. doing nothing is

$$\begin{aligned}\Delta_l^{b,n} &= U_l^b - U_l^n = V_l E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] \\ &= V_l \sum_{d=-1}^{1} \frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3} \\ &= \frac{1}{3} \sum_{d=-1}^{1} \left(V_l \pi_{d+1} - P_{d+1}\right) \\ &= \frac{1}{3} \sum_{d=-1}^{1} \pi_{d+1}\left(V_l - E\left(V_\omega \left| Q = d+1\right.\right)\right)\end{aligned}$$

Since $E\left(V_\omega \left| Q = d+1\right.\right) \geq V_l$ for all d, and when $d = -1$, from Lemma 2.1 we have $E\left(V_\omega \left| Q = 0\right.\right) = \bar{V} > V_l$

$$\Rightarrow \Delta_l^{b,n} < 0$$

The relative payoff from doing nothing vs. selling is

$$\begin{aligned}\Delta_l^{n,s} &= U_l^n - U_l^s = V_l E_D\left[\pi_{D-1}\right] - E_D\left[P_{D-1}\right] \\ &= V_l \sum_{d=-1}^{1} \frac{\pi_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_{d-1}}{3} \\ &= \frac{1}{3} \sum_{d=-1}^{1} \left(V_l \pi_{d-1} - P_{d-1}\right) \\ &= \frac{1}{3} \sum_{d=-1}^{1} \pi_{d-1}\left(V_l - E\left(V_\omega \left| Q = d-1\right.\right)\right)\end{aligned}$$

Since $E\left(V_\omega \left| Q = d-1\right.\right) \geq V_l$ for all d for all d, and when $d = 1$, from Lemma 2.1 we have $E\left(V_\omega \left| Q = 0\right.\right) = \bar{V} > V_l$

$$\Rightarrow \Delta_l^{n,s} < 0$$

In sum, both $\Delta_l^{b,n}$ and $\Delta_l^{n,s}$ are less than zero, which implies that $\Delta_l^{b,s} = \Delta_l^{b,n} + \Delta_l^{n,s}$ is also less than zero, so the negatively informed speculator will always choose to sell. □

Proof of Claim 2.5

Proof. If the initial short position $s = 0$, then the uninformed speculator's payoff of doing nothing will be zero, i.e. $U_{\emptyset}^{n} = 0$. So the relative payoff from buying vs. doing nothing is

$$\begin{aligned}
\Delta_{\emptyset}^{b,n} &= U_{\emptyset}^{b} - U_{\emptyset}^{n} = \bar{V} E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] \\
&= \bar{V} \sum_{d=-1}^{1} \frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3} \\
&= \frac{1}{3} \sum_{d=-1}^{1} \left(\bar{V}\pi_{d+1} - P_{d+1}\right) \\
&= \frac{1}{3} \sum_{d=-1}^{1} \pi_{d+1}\left(\bar{V} - E\left(V_{\omega} \left| Q = d+1\right.\right)\right)
\end{aligned}$$

From Claim 2.3 and Claim 2.4 we have $p_h^b = 1$ and $p_l^s = 1$, so the probability of $Q = 1$ will be

$$\begin{aligned}
\Pr\left(Q = 1\right) &= \frac{1}{3}\left[\left(1-\alpha\right)\left(p_{\emptyset}^{b} + p_{\emptyset}^{n}\right) + \frac{\alpha}{2}\left(p_h^b + p_h^n\right) + \frac{\alpha}{2}\left(p_l^b + p_l^n\right)\right] \\
&= \frac{1}{3}\left[\left(1-\alpha\right)\left(1 - p_{\emptyset}^{s}\right) + \frac{\alpha}{2}\right]
\end{aligned}$$

Thus the expected value of V_{ω} given $Q = 1$ is

$$\begin{aligned}
E\left(V_{\omega} \left| Q = 1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\frac{\alpha}{2}V^{+}\left(1-p_h^s\right) + \frac{\alpha}{2}V^{-}\left(1-p_l^s\right) + \left(1-\alpha\right)\bar{V}\left(1-p_{\emptyset}^{s}\right)\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\frac{\alpha}{2}V^{+} + \left(1-\alpha\right)\bar{V}\left(1-p_{\emptyset}^{s}\right)\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-\alpha\right)\left(1-p_{\emptyset}^{s}\right) + \frac{\alpha}{2}\right]}\left[\frac{\alpha}{2}V^{+} + \left(1-\alpha\right)\bar{V}\left(1-p_{\emptyset}^{s}\right)\right] \\
&> \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-\alpha\right)\left(1-p_{\emptyset}^{s}\right) + \frac{\alpha}{2}\right]}\left[\frac{\alpha}{2}\bar{V} + \left(1-\alpha\right)\bar{V}\left(1-p_{\emptyset}^{s}\right)\right] = \bar{V} > 0
\end{aligned}$$

$$\Rightarrow \pi_1 = 1$$

Similarly, the probability of $Q = 2$ will be

$$\begin{aligned}
\Pr\left(Q = 2\right) &= \frac{1}{3}\left[\left(1-\alpha\right)p_{\emptyset}^{b} + \frac{\alpha}{2}p_h^b + \frac{\alpha}{2}p_l^b\right] \\
&= \frac{1}{3}\left[\left(1-\alpha\right)p_{\emptyset}^{b} + \frac{\alpha}{2}\right]
\end{aligned}$$

So the expected value of V_ω given $Q = 2$ is

$$\begin{aligned}
E\left(V_\omega \,|Q=2\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[\frac{\alpha}{2}V^+ p_h^b + \frac{\alpha}{2}V^- p_l^b + \left(1-\alpha\right)\bar{V} p_\emptyset^b\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[\frac{\alpha}{2}V^+ + \left(1-\alpha\right)\bar{V} p_\emptyset^b\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-\alpha\right)p_\emptyset^b + \frac{\alpha}{2}\right]}\left[\frac{\alpha}{2}V^+ + \left(1-\alpha\right)\bar{V} p_\emptyset^b\right] \\
&> \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-\alpha\right)p_\emptyset^b + \frac{\alpha}{2}\right]}\left[\frac{\alpha}{2}\bar{V} + \left(1-\alpha\right)\bar{V} p_\emptyset^b\right] = \bar{V} > 0
\end{aligned}$$

$$\Rightarrow \pi_2 = 1$$

Finally, from Lemma 2.1 we have $E\left(V_\omega \,|Q=0\right) = \bar{V}$ and $\pi_0 = 1$, thus to sum up

$$E\left(V_\omega \,|Q=0\right) = \bar{V}$$

$$E\left(V_\omega \,|Q=1\right) > \bar{V}$$

$$E\left(V_\omega \,|Q=2\right) > \bar{V}$$

$$\pi_0 = \pi_1 = \pi_2 = 1$$

$$\Rightarrow \Delta_\emptyset^{b,n} < 0$$

Next consider the relative payoff from doing nothing vs. selling, which is

$$\begin{aligned}
\Delta_\emptyset^{n,s} = U_\emptyset^n - U_\emptyset^s &= \bar{V} E_D\left[\pi_{D-1}\right] - E_D\left[P_{D-1}\right] \\
&= \bar{V}\sum_{d=-1}^{1}\frac{\pi_{d-1}}{3} - \sum_{d=-1}^{1}\frac{P_{d-1}}{3} \\
&= \frac{1}{3}\sum_{d=-1}^{1}\left(\bar{V}\pi_{d-1} - P_{d-1}\right) \\
&= \frac{1}{3}\sum_{d=-1}^{1}\pi_{d-1}\left(\bar{V} - E\left(V_\omega \,|Q=d-1\right)\right)
\end{aligned}$$

From Claim 2.3 we have $p_h^b = 1$, thus $p_h^s = 0$. According to Lemma 2.2, since $p_h^b = 1 > \frac{2\bar{V}}{\alpha V^+}$, then $E\left(V_\omega \,|Q=-1\right) < 0$, firm will not invest when $Q = -1$, i.e. $\pi_{-1} = 0$. According to Lemma 2.3, since $p_h^s = 0 < 1 - \frac{2\bar{V}}{\alpha V^+}$, then $E\left(V_\omega \,|Q=-2\right) < 0$, firm will not invest when $Q=-2$, i.e. $\pi_{-2} = 0$. Finally from Lemma 2.1 we have $E\left(V_\omega \,|Q=0\right) = \bar{V}$, thus

$$\Rightarrow \Delta_{\emptyset}^{n,s} = 0$$

In sum, we have $\Delta_{\emptyset}^{b,n} < 0$ and $\Delta_{\emptyset}^{n,s} = 0$, which implies that $\Delta_{\emptyset}^{b,s} = \Delta_{\emptyset}^{b,n} + \Delta_{\emptyset}^{n,s} < 0$. So the uninformed speculator will never choose to buy, he will mix between doing nothing and selling. □

Proof of Claim 2.6

Proof. When the initial short position of speculator is not revealed, firm and market maker will assume the speculator initially has no position on the stock, which is $s = 0$. From Claim 2.3, 2.4, and 2.5 we have, positively informed speculator will always buy; negatively informed speculator will always sell; uninformed speculator will mix between doing nothing and selling. That is

$$p_h^b = 1, p_h^s = p_h^n = 0$$

$$p_l^s = 1, p_l^b = p_l^n = 0$$

$$p_{\emptyset}^b = 0, p_{\emptyset}^s + p_{\emptyset}^n = 1$$

According to Lemma 2.2, since $p_h^b = 1 > \frac{2\bar{V}}{\alpha V^+}$, then $E\left(V_\omega \,|Q=-1\right) < 0$, firm will not invest when $Q=-1$, and $P_{-1} = 0$.

According to Lemma 2.3, since $p_h^s = 0 < 1 - \frac{2\bar{V}}{\alpha V^+}$, then $E\left(V_\omega \,|Q=-2\right) < 0$, firm will not invest when $Q=-2$, and $P_{-2} = 0$.

According to Lemma 2.1 we have $E\left(V_\omega \,|Q=0\right) = \bar{V}$, firm will invest when $Q=0$, and $P_0 = \bar{V}$. Next consider the cases when $Q = 1, 2$.

The probability of $Q = 2$ will be

$$\begin{aligned}\Pr\left(Q=2\right) &= \frac{1}{3}\left[\left(1-\alpha\right)p_{\emptyset}^b + \frac{\alpha}{2}p_h^b + \frac{\alpha}{2}p_l^b\right] \\ &= \frac{1}{3}\left[\frac{\alpha}{2}\right]\end{aligned}$$

Then the expected value of V_ω given $Q = 2$ is

$$\begin{aligned} E\left(V_\omega \left| Q=2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[\frac{\alpha}{2}V^{+}p_h^b + \frac{\alpha}{2}V^{-}p_l^b + (1-\alpha)\,\bar{V}p_\emptyset^b\right] \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[\frac{\alpha}{2}V^{+}\right] \\ &= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}\right]}\left[\frac{\alpha}{2}V^{+}\right] = V^{+} > 0 \end{aligned}$$

So the firm will invest when $Q = 2$, and the market maker will set price to equal V^{+}

Similarly, the probability of $Q = 1$ will be

$$\begin{aligned} \Pr\left(Q=1\right) &= \frac{1}{3}\left[(1-\alpha)\left(p_\emptyset^b + p_\emptyset^n\right) + \frac{\alpha}{2}\left(p_h^b + p_h^n\right) + \frac{\alpha}{2}\left(p_l^b + p_l^n\right)\right] \\ &= \frac{1}{3}\left[(1-\alpha)\,p_\emptyset^n + \frac{\alpha}{2}\right] \end{aligned}$$

Thus the expected value of V_ω given $Q = 1$ will be

$$\begin{aligned} E\left(V_\omega \left| Q=1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\frac{\alpha}{2}V^{+}\left(1-p_h^s\right) + \frac{\alpha}{2}V^{-}\left(1-p_l^s\right) + (1-\alpha)\,\bar{V}\left(1-p_\emptyset^s\right)\right] \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\frac{\alpha}{2}V^{+} + (1-\alpha)\,\bar{V}p_\emptyset^n\right] \\ &= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2} + (1-\alpha)\,p_\emptyset^n\right]}\left[\frac{\alpha}{2}V^{+} + (1-\alpha)\,\bar{V}p_\emptyset^n\right] \\ &= \frac{\frac{\alpha}{2}V^{+} + p_\emptyset^n\,(1-\alpha)\,\bar{V}}{\frac{\alpha}{2} + p_\emptyset^n\,(1-\alpha)} > 0 \end{aligned}$$

So the firm will invest when $Q = 1$, and the market maker will set price to equal $\frac{\frac{\alpha}{2}V^{+}+p_\emptyset^n(1-\alpha)\bar{V}}{\frac{\alpha}{2}+p_\emptyset^n(1-\alpha)}$. □

Proof of Claim 2.7

Proof. If the initial short position $s > 0$, then the negatively informed speculator's relative payoff from buying vs. doing nothing will be

$$\begin{aligned}
\Delta_l^{b,n} &= U_l^b - U_l^n = V_l E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] + s\left(E_D\left[P_D\right] - E_D\left[P_{D+1}\right]\right) \\
&= \left[V_l \sum_{d=-1}^{1} \frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3}\right] + s\left[\sum_{d=-1}^{1} \frac{P_d}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\left[V_l \pi_{d+1} - P_{d+1}\right] + \frac{s}{3}\sum_{d=-1}^{1}\left[P_d - P_{d+1}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1} \pi_{d+1}\left[V_l - E\left(V_\omega \left| Q = d+1\right.\right)\right] + \frac{s}{3}\left[P_{-1} - P_2\right]
\end{aligned}$$

Since the first term $\frac{1}{3}\sum_{d=-1}^{1} \pi_{d+1}\left[V_l - E\left(V_\omega \left| Q = d+1\right.\right)\right]$ is less than zero, which is proved in Claim 2.4. And from Claim 2.6, we have $P_{-1} = 0$, $P_2 = V^+$, which means the second term $\frac{s}{3}\left[P_{-1} - P_2\right]$ is also less than zero. Therefore

$$\Rightarrow \Delta_l^{b,n} < 0$$

The negatively informed speculator's relative payoff from doing nothing vs. selling is

$$\begin{aligned}
\Delta_l^{n,s} &= U_l^n - U_l^s = V_l E_D\left[\pi_{D-1}\right] - E_D\left[P_{D-1}\right] + s\left(E_D\left[P_{D-1}\right] - E_D\left[P_D\right]\right) \\
&= \left[V_l \sum_{d=-1}^{1} \frac{\pi_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_{d-1}}{3}\right] + s\left[\sum_{d=-1}^{1} \frac{P_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_d}{3}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\left[V_l \pi_{d-1} - P_{d-1}\right] + \frac{s}{3}\sum_{d=-1}^{1}\left[P_{d-1} - P_d\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1} \pi_{d-1}\left[V_l - E\left(V_\omega \left| Q = d-1\right.\right)\right] + \frac{s}{3}\left[P_{-2} - P_1\right]
\end{aligned}$$

Since the first term $\frac{1}{3}\sum_{d=-1}^{1} \pi_{d-1}\left[V_l - E\left(V_\omega \left| Q = d-1\right.\right)\right]$ is less than zero, which is proved in Claim 2.4. And from Claim 2.6, we have $P_{-2} = 0$, $P_1 = \frac{\frac{\alpha}{2}V^+ + p_\emptyset^n(1-\alpha)\bar{V}}{\frac{\alpha}{2} + p_\emptyset^n(1-\alpha)}$, which means the second term $\frac{s}{3}\left[P_{-2} - P_1\right]$ is also less than zero. Therefore

$$\Rightarrow \Delta_l^{n,s} < 0$$

In sum, both $\Delta_l^{b,n}$ and $\Delta_l^{n,s}$ are less than zero, which implies that $\Delta_l^{b,s} = \Delta_l^{b,n} + \Delta_l^{n,s}$ is also less than zero, so the negatively informed speculator will always choose to sell. $\square$

Proof of Claim 2.8

Proof. If the initial short position $s > 0$, then the uninformed speculator's relative payoff from buying vs. doing nothing will be

$$\begin{aligned}
\Delta_{\emptyset}^{b,n} &= U_{\emptyset}^{b} - U_{\emptyset}^{n} = \bar{V} E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] + s\left(E_D\left[P_D\right] - E_D\left[P_{D+1}\right]\right) \\
&= \left[\bar{V}\sum_{d=-1}^{1}\frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1}\frac{P_{d+1}}{3}\right] + s\left[\sum_{d=-1}^{1}\frac{P_d}{3} - \sum_{d=-1}^{1}\frac{P_{d+1}}{3}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\left[\bar{V}\pi_{d+1} - P_{d+1}\right] + \frac{s}{3}\sum_{d=-1}^{1}\left[P_d - P_{d+1}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\pi_{d+1}\left[\bar{V} - E\left(V_\omega \,|Q = d+1\right)\right] + \frac{s}{3}\left[P_{-1} - P_2\right]
\end{aligned}$$

Since the first term $\frac{1}{3}\sum_{d=-1}^{1}\pi_{d+1}\left[\bar{V} - E\left(V_\omega \,|Q = d+1\right)\right]$ is less than zero, which is proved in Claim 2.5. And from Claim 2.6, we have $P_{-1} = 0$, $P_2 = V^+$, which means the second term $\frac{s}{3}\left[P_{-1} - P_2\right]$ is also less than zero. Therefore

$$\Rightarrow \Delta_{\emptyset}^{b,n} < 0$$

The uninformed speculator's relative payoff from doing nothing vs. selling is

$$\begin{aligned}
\Delta_{\emptyset}^{n,s} &= U_{\emptyset}^{n} - U_{\emptyset}^{s} = \bar{V} E_D\left[\pi_{D-1}\right] - E_D\left[P_{D-1}\right] + s\left(E_D\left[P_{D-1}\right] - E_D\left[P_D\right]\right) \\
&= \left[\bar{V}\sum_{d=-1}^{1}\frac{\pi_{d-1}}{3} - \sum_{d=-1}^{1}\frac{P_{d-1}}{3}\right] + s\left[\sum_{d=-1}^{1}\frac{P_{d-1}}{3} - \sum_{d=-1}^{1}\frac{P_d}{3}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\left[\bar{V}\pi_{d-1} - P_{d-1}\right] + \frac{s}{3}\sum_{d=-1}^{1}\left[P_{d-1} - P_d\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\pi_{d-1}\left[\bar{V} - E\left(V_\omega \,|Q = d-1\right)\right] + \frac{s}{3}\left[P_{-2} - P_1\right]
\end{aligned}$$

Since the first term $\frac{1}{3}\sum_{d=-1}^{1}\pi_{d-1}\left[\bar{V} - E\left(V_\omega \,|Q = d-1\right)\right]$ is equal to zero, which is proved in Claim 2.5. And from Claim 2.6, we have $P_{-2} = 0$, $P_1 = \frac{\frac{\alpha}{2}V^+ + p_{\emptyset}^{n}(1-\alpha)\bar{V}}{\frac{\alpha}{2} + p_{\emptyset}^{n}(1-\alpha)}$, which means the second term $\frac{s}{3}\left[P_{-2} - P_1\right]$ is less than zero. Therefore

$$\Rightarrow \Delta_{\emptyset}^{n,s} < 0$$

In sum, both $\Delta_{\emptyset}^{b,n}$ and $\Delta_{\emptyset}^{n,s}$ are less than zero, which implies that $\Delta_{\emptyset}^{b,s} = \Delta_{\emptyset}^{b,n} + \Delta_{\emptyset}^{n,s}$ is also less than zero, so the uninformed speculator will always choose to sell. □

Proof of Claim 2.9

Proof. If the initial short position $s > 0$, then the positively informed speculator's relative payoff from buying vs. doing nothing will be

$$\begin{aligned}
\Delta_h^{b,n} = U_h^b - U_h^n &= V_h E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] + s\left(E_D\left[P_D\right] - E_D\left[P_{D+1}\right]\right) \\
&= \left[V_h \sum_{d=-1}^{1} \frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3}\right] + s\left[\sum_{d=-1}^{1} \frac{P_d}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3}\right] \\
&= \frac{1}{3} \sum_{d=-1}^{1} \left[V_h \pi_{d+1} - P_{d+1}\right] + \frac{s}{3} \sum_{d=-1}^{1} \left[P_d - P_{d+1}\right] \\
&= \frac{1}{3} \sum_{d=-1}^{1} \pi_{d+1}\left[V_h - E\left(V_\omega \left| Q = d+1\right.\right)\right] + \frac{s}{3}\left[P_{-1} - P_2\right]
\end{aligned}$$

From Lemma 2.1, 2.4 and 2.5 we have $\pi_0 = \pi_1 = \pi_2 = 1$,

$$\Rightarrow \Delta_h^{b,n} = \frac{1}{3}\left[3V_h - P_0 - P_1 - P_2\right] + \frac{s}{3}\left[P_{-1} - P_2\right]$$

Also Claim 2.6 gives us $P_{-1} = 0$, $P_0 = \bar{V}$, $P_2 = V^+$, and $P_1 = \frac{\frac{\alpha}{2}V^+ + p_{\emptyset}^n(1-\alpha)\bar{V}}{\frac{\alpha}{2} + p_{\emptyset}^n(1-\alpha)}$. According to Claim 2.8, we can see $p_{\emptyset}^n = 0$, thus $P_{-1} = V^+$.

$$\Rightarrow \Delta_h^{b,n} = \frac{1}{3}\left[V^+ - \bar{V}\right] - \frac{s}{3}V^+$$

Therefore we have

$$\begin{aligned}
\Delta_h^{b,n} > 0 &\Leftrightarrow s < 1 - \frac{\bar{V}}{V^+} \\
\Delta_h^{b,n} < 0 &\Leftrightarrow s > 1 - \frac{\bar{V}}{V^+}
\end{aligned}$$

Similarly the positively informed speculator's relative payoff from doing nothing vs. selling is

$$\begin{aligned}
\Delta_h^{n,s} &= U_h^n - U_h^s = V_h E_D\left[\pi_{D-1}\right] - E_D\left[P_{D-1}\right] + s\left(E_D\left[P_{D-1}\right] - E_D\left[P_D\right]\right) \\
&= \left[V_h \sum_{d=-1}^{1} \frac{\pi_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_{d-1}}{3}\right] + s\left[\sum_{d=-1}^{1} \frac{P_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_d}{3}\right] \\
&= \frac{1}{3} \sum_{d=-1}^{1} \left[V_h \pi_{d-1} - P_{d-1}\right] + \frac{s}{3} \sum_{d=-1}^{1} \left[P_{d-1} - P_d\right] \\
&= \frac{1}{3} \sum_{d=-1}^{1} \pi_{d-1}\left[V_h - E\left(V_\omega \left| Q = d-1\right.\right)\right] + \frac{s}{3}\left[P_{-2} - P_1\right]
\end{aligned}$$

From Claim 2.6 we have $\pi_0 = 1$, and $\pi_{-1} = \pi_{-2} = 0$

$$\begin{aligned}
\Rightarrow \Delta_h^{n,s} &= \frac{1}{3}\left[V_h - P_0\right] + \frac{s}{3}\left[P_{-2} - P_1\right] \\
&= \frac{1}{3}\left[V^+ - \bar{V}\right] - \frac{s}{3} V^+
\end{aligned}$$

which gives us

$$\begin{aligned}
\Delta_h^{n,s} > 0 &\Leftrightarrow s < 1 - \frac{\bar{V}}{V^+} \\
\Delta_h^{n,s} < 0 &\Leftrightarrow s > 1 - \frac{\bar{V}}{V^+}
\end{aligned}$$

In sum, we have

- If $s < 1 - \frac{\bar{V}}{V^+}$, then $\Delta_h^{b,b} > 0$ and $\Delta_h^{n,s} > 0$, which implies that $\Delta_h^{b,s} = \Delta_h^{b,n} + \Delta_h^{n,s} > 0$. So the positively informed speculator will always choose to buy.

- If $s > 1 - \frac{\bar{V}}{V^+}$, then $\Delta_h^{b,b} < 0$ and $\Delta_h^{n,s} < 0$, which implies that $\Delta_h^{b,s} = \Delta_h^{b,n} + \Delta_h^{n,s} < 0$. So the positively informed speculator will always choose to sell.

As for $s = 1 - \frac{\bar{V}}{V^+}$, with all prices given constant, it will be non-generic in this case.

□

Proof of Claim 2.10

Proof. If the initial short position $s > 0$ is revealed, then the negatively informed speculator's relative payoff from buying vs. doing nothing will be

$$\begin{aligned}
\Delta_l^{b,n} &= U_l^b - U_l^n = V_l E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] + s\left(E_D\left[P_D\right] - E_D\left[P_{D+1}\right]\right) \\
&= \left[V_l \sum_{d=-1}^{1} \frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3}\right] + s\left[\sum_{d=-1}^{1} \frac{P_d}{3} - \sum_{d=-1}^{1} \frac{P_{d+1}}{3}\right] \\
&= \frac{1}{3} \sum_{d=-1}^{1} \left[V_l \pi_{d+1} - P_{d+1}\right] + \frac{s}{3} \sum_{d=-1}^{1} \left[P_d - P_{d+1}\right] \\
&= \frac{1}{3} \sum_{d=-1}^{1} \pi_{d+1}\left[V_l - E\left(V_\omega \left| Q = d+1\right.\right)\right] + \frac{s}{3}\left[P_{-1} - P_2\right]
\end{aligned}$$

Since the first term $\frac{1}{3} \sum\limits_{d=-1}^{1} \pi_{d+1}\left[V_l - E\left(V_\omega \left| Q = d+1\right.\right)\right]$ is less than zero, which is proved in Claim 2.4, so next we will discuss the sign of the second term.

$$\frac{s}{3}\left[P_{-1} - P_2\right] = \frac{s}{3}\left[\pi_{-1} E\left(V_\omega \left| Q = -1\right.\right) - \pi_2 E\left(V_\omega \left| Q = 2\right.\right)\right]$$

Case 1. If $p_h^b > \frac{2\bar{V}}{\alpha V^+}$, then by Lemma 2 we will have $E\left(V_\omega \left| Q = -1\right.\right) < 0$ and $P_{-1} = 0$, and by Lemma 2.5 we have $E\left(V_\omega \left| Q = 2\right.\right) > 0$ and $P_2 > 0$. Thus the second term $\frac{s}{3}\left[P_{-1} - P_2\right]$ is less than zero.

Case 2. If $0 < p_h^b \leq \frac{2\bar{V}}{\alpha V^+}$, then by SCP we have $p_\emptyset^b = p_l^b = 0$.

The expected value of V_ω given $Q = 2$ then will be

$$\begin{aligned}
E\left(V_\omega \left| Q = 2\right.\right) &= \frac{1}{3} \frac{1}{\Pr\left(Q = 2\right)}\left[p_h^b \frac{\alpha}{2} V^+ + p_l^b \frac{\alpha}{2} V^- + p_\emptyset^b\left(1 - \alpha\right)\bar{V}\right] \\
&= \frac{1}{3} \frac{1}{\Pr\left(Q = 2\right)}\left[p_h^b \frac{\alpha}{2} V^+\right] \\
&= \frac{1}{3} \frac{1}{\frac{1}{3}\left[\frac{\alpha}{2} p_h^b\right]}\left[p_h^b \frac{\alpha}{2} V^+\right] = V^+
\end{aligned}$$

The expected value of V_ω given $Q = -1$ then will be

$$\begin{aligned}
E\left(V_\omega \left| Q = -1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \left(1-p_l^b\right)\frac{\alpha}{2}V^- + \left(1-p_\emptyset^b\right)\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-p_h^b\right)\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{\left[\bar{V}-p_h^b\frac{\alpha}{2}V^+\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]} < \frac{\left[V^+-p_h^b\frac{\alpha}{2}V^+\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]} = V^+
\end{aligned}$$

Thus we have

$$\begin{aligned}
\frac{s}{3}\left[P_{-1}-P_2\right] &= \frac{s}{3}\left[\pi_{-1}E\left(V_\omega \left| Q=-1\right.\right) - \pi_2 E\left(V_\omega \left| Q=2\right.\right)\right] \\
&= \frac{s}{3}\left[\pi_{-1}E\left(V_\omega \left| Q=-1\right.\right) - V^+\right] \\
&= \frac{s}{3}\left[\pi_{-1}\left[E\left(V_\omega \left| Q=-1\right.\right) - V^+\right] - \left(1-\pi_{-1}\right)V^+\right] < 0
\end{aligned}$$

Case 3. If $p_h^b = 0$, then by SCP we have $p_\emptyset^b = p_l^b = 0$. In this case all three types speculators will not choose to buy, which leaves $Q = 2$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is positively informed, thus the expected value of V_ω will be $E\left(V_\omega \left| Q=2\right.\right) = V^+$.

Meanwhile, the expected value of V_ω given $Q = -1$ is

$$\begin{aligned}
E\left(V_\omega \left| Q = -1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \left(1-p_l^b\right)\frac{\alpha}{2}V^- + \left(1-p_\emptyset^b\right)\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \bar{V} < V^+
\end{aligned}$$

Thus we have

$$\begin{aligned}
\frac{s}{3}\left[P_{-1}-P_2\right] &= \frac{s}{3}\left[\pi_{-1}E\left(V_\omega \left| Q=-1\right.\right) - \pi_2 E\left(V_\omega \left| Q=2\right.\right)\right] \\
&= \frac{s}{3}\left[\bar{V}-V^+\right] < 0
\end{aligned}$$

In sum $\frac{s}{3}\left[P_{-1}-P_2\right]$ is less than zero in all three cases, therefore

$$\Rightarrow \Delta_l^{b,n}<0$$

Then we consider the negatively informed speculator's relative payoff from doing nothing vs. selling, which is

$$\begin{aligned}
\Delta_l^{n,s} &= U_l^n-U_l^s=V_lE_D\left[\pi_{D-1}\right]-E_D\left[P_{D-1}\right]+s\left(E_D\left[P_{D-1}\right]-E_D\left[P_D\right]\right)\\
&=\left[V_l\sum_{d=-1}^{1}\frac{\pi_{d-1}}{3}-\sum_{d=-1}^{1}\frac{P_{d-1}}{3}\right]+s\left[\sum_{d=-1}^{1}\frac{P_{d-1}}{3}-\sum_{d=-1}^{1}\frac{P_d}{3}\right]\\
&=\frac{1}{3}\sum_{d=-1}^{1}\left[V_l\pi_{d-1}-P_{d-1}\right]+\frac{s}{3}\sum_{d=-1}^{1}\left[P_{d-1}-P_d\right]\\
&=\frac{1}{3}\sum_{d=-1}^{1}\pi_{d-1}\left[V_l-E\left(V_\omega\left|Q=d-1\right.\right)\right]+\frac{s}{3}\left[P_{-2}-P_1\right]
\end{aligned}$$

Since the first term $\frac{1}{3}\sum\limits_{d=-1}^{1}\pi_{d-1}\left[V_l-E\left(V_\omega\left|Q=d-1\right.\right)\right]$ is less than zero, which is proved in Claim 2.4, so next we will discuss the sign of the second term.

$$\frac{s}{3}\left[P_{-2}-P_1\right]=\frac{s}{3}\left[\pi_{-2}E\left(V_\omega\left|Q=-2\right.\right)-\pi_1E\left(V_\omega\left|Q=1\right.\right)\right]$$

Case 1. If $p_h^s<1-\frac{2\bar{V}}{\alpha V^+}$, then by Lemma 3 we will have $E\left(V_\omega\left|Q=-2\right.\right)<0$ and $P_{-2}=0$, and by Lemma 2.4 we have $E\left(V_\omega\left|Q=1\right.\right)>0$ and $P_1>0$. Thus the second term $\frac{s}{3}\left[P_{-2}-P_1\right]$ is less than zero.

Case 2. If $1-\frac{2\bar{V}}{\alpha V^+}\leq p_h^s<1$, then by SCP we have $p_\emptyset^s=p_l^s=1$.

The expected value of V_ω given $Q=1$ then will be

$$\begin{aligned}
E\left(V_\omega\left|Q=1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^+ +\left(1-p_l^s\right)\frac{\alpha}{2}V^- +\left(1-p_\emptyset^s\right)\left(1-\alpha\right)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^+\right]\\
&=\frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}\left(1-p_h^s\right)\right]}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^+\right]=V^+
\end{aligned}$$

The expected value of V_ω given $Q=-2$ then will be

$$\begin{aligned}
E\left(V_\omega\left|Q=-2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + p_l^s\frac{\alpha}{2}V^- + p_\emptyset^s\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[p_h^s\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[p_h^s\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{\left[\bar{V}-\left(1-p_h^s\right)\frac{\alpha}{2}V^+\right]}{\left[1-\left(1-p_h^s\right)\frac{\alpha}{2}\right]} < \frac{\left[V^+-\left(1-p_h^s\right)\frac{\alpha}{2}V^+\right]}{\left[1-\left(1-p_h^s\right)\frac{\alpha}{2}\right]} = V^+
\end{aligned}$$

Thus we have

$$\begin{aligned}
\frac{s}{3}\left[P_{-2}-P_1\right] &= \frac{s}{3}\left[\pi_{-2}E\left(V_\omega\left|Q=-2\right.\right)-\pi_1E\left(V_\omega\left|Q=1\right.\right)\right] \\
&= \frac{s}{3}\left[\pi_{-2}E\left(V_\omega\left|Q=-2\right.\right)-V^+\right] \\
&= \frac{s}{3}\left[\pi_{-2}\left[E\left(V_\omega\left|Q=-2\right.\right)-V^+\right]-\left(1-\pi_{-2}\right)V^+\right] < 0
\end{aligned}$$

Case 3. If $p_h^s=1$, then by SCP we have $p_\emptyset^s=p_l^s=1$. In this case all three types speculators will choose to sell, which leaves $Q=1$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is positively informed, thus the expected value of V_ω will be $E\left(V_\omega\left|Q=1\right.\right)=V^+$.

Meanwhile, the expected value of V_ω given $Q=-2$ is

$$\begin{aligned}
E\left(V_\omega\left|Q=-2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + p_l^s\frac{\alpha}{2}V^- + p_\emptyset^s\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \bar{V} < V^+
\end{aligned}$$

Thus we have

$$\begin{aligned}
\frac{s}{3}\left[P_{-2}-P_1\right] &= \frac{s}{3}\left[\pi_{-2}E\left(V_\omega\left|Q=-2\right.\right)-\pi_1E\left(V_\omega\left|Q=1\right.\right)\right] \\
&= \frac{s}{3}\left[\bar{V}-V^+\right] < 0
\end{aligned}$$

In sum $\frac{s}{3}\left[P_{-2}-P_{1}\right]$ is less than zero in all three cases, therefore

$$\Rightarrow \Delta_{l}^{n,s}<0$$

To sum up, both $\Delta_{l}^{b,n}$ and $\Delta_{l}^{n,s}$ are less than zero, which implies that $\Delta_{l}^{b,s}=\Delta_{l}^{b,n}+\Delta_{l}^{n,s}$ is also less than zero, so the negatively informed speculator will always choose to sell. $\square$

Proof of Claim 2.11

Proof. If the initial short position $s>0$ is revealed, then the uninformed speculator's relative payoff from buying vs. doing nothing will be

$$\begin{aligned}
\Delta_{\emptyset}^{b,n} &= U_{\emptyset}^{b}-U_{\emptyset}^{n}=\bar{V}E_{D}\left[\pi_{D+1}\right]-E_{D}\left[P_{D+1}\right]+s\left(E_{D}\left[P_{D}\right]-E_{D}\left[P_{D+1}\right]\right) \\
&= \left[\bar{V}\sum_{d=-1}^{1}\frac{\pi_{d+1}}{3}-\sum_{d=-1}^{1}\frac{P_{d+1}}{3}\right]+s\left[\sum_{d=-1}^{1}\frac{P_{d}}{3}-\sum_{d=-1}^{1}\frac{P_{d+1}}{3}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\left[\bar{V}\pi_{d+1}-P_{d+1}\right]+\frac{s}{3}\sum_{d=-1}^{1}\left[P_{d}-P_{d+1}\right] \\
&= \frac{1}{3}\sum_{d=-1}^{1}\pi_{d+1}\left[\bar{V}-E\left(V_{\omega}\left|Q=d+1\right.\right)\right]+\frac{s}{3}\left[P_{-1}-P_{2}\right]
\end{aligned}$$

Since the second term $\frac{s}{3}\left[P_{-1}-P_{2}\right]$ is less than zero, which is proved in Claim 10, so next we will discuss the sign of the first term.

First, consider the expected value of V_{ω} given $Q=1$, which is

$$E\left(V_{\omega}\left|Q=1\right.\right)=\frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\left(1-p_{h}^{s}\right)\frac{\alpha}{2}V^{+}+\left(1-p_{l}^{s}\right)\frac{\alpha}{2}V^{-}+\left(1-p_{\emptyset}^{s}\right)\left(1-\alpha\right)\bar{V}\right]$$

According to Claim 2.10, we have $p_{l}^{s}=1$, thus

$$E\left(V_{\omega}\left|Q=1\right.\right)=\frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\left(1-p_{h}^{s}\right)\frac{\alpha}{2}V^{+}+\left(1-p_{\emptyset}^{s}\right)\left(1-\alpha\right)\bar{V}\right]$$

Case 1. If $p_{h}^{s}<1$, then the probability of $Q=1$ will be

$$\Pr\left(Q=1\right)=\frac{1}{3}\left[\left(1-p_{h}^{s}\right)\frac{\alpha}{2}+\left(1-p_{\emptyset}^{s}\right)\left(1-\alpha\right)\right]>0$$

Therefore

$$E\left(V_\omega \left| Q=1\right.\right)=\frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-p_h^s\right)\frac{\alpha}{2}+\left(1-p_\emptyset^s\right)(1-\alpha)\right]}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^+ +\left(1-p_\emptyset^s\right)(1-\alpha)\bar{V}\right]$$
$$>\frac{1}{\left[\left(1-p_h^s\right)\frac{\alpha}{2}+\left(1-p_\emptyset^s\right)(1-\alpha)\right]}\left[\left(1-p_h^s\right)\frac{\alpha}{2}\bar{V}+\left(1-p_\emptyset^s\right)(1-\alpha)\bar{V}\right]=\bar{V}$$

Case 2. If $p_h^s=1$, then by SCP we have $p_\emptyset^s=1$. In this case all three types speculators will choose to sell, which leaves $Q=1$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is positively informed, thus the expected value of V_ω will be $E\left(V_\omega \left| Q=1\right.\right)=V^+>\bar{V}$.

In sum, $E\left(V_\omega \left| Q=1\right.\right)$ is greater than $\bar{V}$ in both cases.

Second, consider the expected value of V_ω given $Q=2$, which is

$$E\left(V_\omega \left| Q=2\right.\right)=\frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_h^b\frac{\alpha}{2}V^+ +p_l^b\frac{\alpha}{2}V^- +p_\emptyset^b(1-\alpha)\bar{V}\right]$$

According to Claim 2.10, we have $p_l^b=0$, thus

$$E\left(V_\omega \left| Q=2\right.\right)=\frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_h^b\frac{\alpha}{2}V^+ +p_\emptyset^b(1-\alpha)\bar{V}\right]$$

Case 1. If $p_h^b>0$, then the probability of $Q=2$ will be

$$\Pr\left(Q=2\right)=\frac{1}{3}\left[p_h^b\frac{\alpha}{2}+p_\emptyset^b(1-\alpha)\right]>0$$

Therefore

$$E\left(V_\omega \left| Q=2\right.\right)=\frac{1}{3}\frac{1}{\frac{1}{3}\left[p_h^b\frac{\alpha}{2}+p_\emptyset^b(1-\alpha)\right]}\left[p_h^b\frac{\alpha}{2}V^+ +p_\emptyset^b(1-\alpha)\bar{V}\right]$$
$$>\frac{1}{\left[p_h^b\frac{\alpha}{2}+p_\emptyset^b(1-\alpha)\right]}\left[p_h^b\frac{\alpha}{2}\bar{V}+p_\emptyset^b(1-\alpha)\bar{V}\right]=\bar{V}$$

Case 2. If $p_h^b=0$, then by SCP we have $p_\emptyset^b=0$. In this case all three types speculators will not choose to buy, which leaves $Q=2$ to be an out-of-equilibrium trading result. Then D1 implies that the deviator is positively informed, thus the expected value of V_ω will be $E\left(V_\omega \left| Q=2\right.\right)=V^+>\bar{V}$.

In sum, $E\left(V_\omega \left| Q=2\right.\right)$ is greater than $\bar{V}$ in both cases.

Finally, from Lemma 2.1 we have $E\left(V_\omega \,|Q=0\right)=\bar{V}$, thus

$$E\left(V_\omega \,|Q=2\right)>\bar{V}>0$$

$$E\left(V_\omega \,|Q=1\right)>\bar{V}>0$$

$$E\left(V_\omega \,|Q=0\right)=\bar{V}>0$$

$$\Rightarrow \frac{1}{3}\sum_{d=-1}^{1}\pi_{d+1}\left[\bar{V}-E\left(V_\omega \,|Q=d+1\right)\right]<0$$

So we have

$$\Rightarrow \Delta_{\emptyset}^{b,n}<0$$

Hence, when the initial short position $s>0$ is revealed, the uninformed speculator will never choose to buy. □

Proof of Claim 2.12

Proof. When the initial short position $s>0$ is revealed, from Claim 2.10 and 2.11 we have, $p_l^s=1$ and $p_{\emptyset}^b=0$.

First, if $p_h^s>0$, then by SCP we will have $p_{\emptyset}^s=1$.

Second, if $p_h^s=0$, then by Lemma 2.3 we have $E\left(V_\omega \,|Q=-2\right)<0$, so the firm will not invest when $Q=-2$, i.e. $\pi_{-2}=0$. The uninformed speculator's relative payoff from doing nothing vs. selling will be

$$\begin{aligned}
\Delta_{\emptyset}^{n,s} &= U_{\emptyset}^{n}-U_{\emptyset}^{s}=\bar{V}E_D\left[\pi_{D-1}\right]-E_D\left[P_{D-1}\right]+s\left(E_D\left[P_{D-1}\right]-E_D\left[P_D\right]\right)\\
&=\left[\bar{V}\sum_{d=-1}^{1}\frac{\pi_{d-1}}{3}-\sum_{d=-1}^{1}\frac{P_{d-1}}{3}\right]+s\left[\sum_{d=-1}^{1}\frac{P_{d-1}}{3}-\sum_{d=-1}^{1}\frac{P_d}{3}\right]\\
&=\frac{1}{3}\sum_{d=-1}^{1}\left[\bar{V}\pi_{d-1}-P_{d-1}\right]+\frac{s}{3}\sum_{d=-1}^{1}\left[P_{d-1}-P_d\right]\\
&=\frac{1}{3}\sum_{d=-1}^{1}\pi_{d-1}\left[\bar{V}-E\left(V_\omega \,|Q=d-1\right)\right]+\frac{s}{3}\left[P_{-2}-P_1\right]\\
&=\frac{1}{3}\sum_{d=-1}^{1}\pi_{d-1}\left[\bar{V}-E\left(V_\omega \,|Q=d-1\right)\right]+\frac{s}{3}\left[P_{-2}-P_1\right]
\end{aligned}$$

From Lemma 2.1 we have $E\left(V_\omega\left|Q=0\right.\right)=\bar{V}$, and from Lemma 2.3 we have $P_{-2}=0$, thus

$$\Delta_{\emptyset}^{n,s}=\frac{1}{3}\left[\pi_{-1}\bar{V}-P_{-1}-sP_1\right]$$

Then we solve for the price range of P_1 and P_{-1}.

Since The expected value of V_ω given $Q=1$ is

$$\begin{aligned}
E\left(V_\omega\left|Q=1\right.\right)&=\frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^{+}+\left(1-p_l^s\right)\frac{\alpha}{2}V^{-}+\left(1-p_{\emptyset}^s\right)\left(1-\alpha\right)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\frac{\alpha}{2}V^{+}+\left(1-p_{\emptyset}^s\right)\left(1-\alpha\right)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}+\left(1-p_{\emptyset}^s\right)\left(1-\alpha\right)\right]}\left[\frac{\alpha}{2}V^{+}+\left(1-p_{\emptyset}^s\right)\left(1-\alpha\right)\bar{V}\right]\\
&>\frac{1}{\left[\frac{\alpha}{2}+\left(1-p_{\emptyset}^s\right)\left(1-\alpha\right)\right]}\left[\frac{\alpha}{2}\bar{V}+\left(1-p_{\emptyset}^s\right)\left(1-\alpha\right)\bar{V}\right]=\bar{V}
\end{aligned}$$

So we have the range of price when $Q=1$ will be $P_1\in(\bar{V},V^{+}]$.

Also, the expected value of V_ω given $Q=-1$ is

$$\begin{aligned}
E\left(V_\omega\left|Q=-1\right.\right)&=\frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^{+}+\left(1-p_l^b\right)\frac{\alpha}{2}V^{-}+\left(1-p_{\emptyset}^b\right)\left(1-\alpha\right)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^{+}+\frac{\alpha}{2}V^{-}+\left(1-\alpha\right)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-p_h^b\right)\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^{+}+\frac{\alpha}{2}V^{-}+\left(1-\alpha\right)\bar{V}\right]\\
&=\frac{\left[\bar{V}-p_h^b\frac{\alpha}{2}V^{+}\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]}
\end{aligned}$$

Let

$$V\left(p_h^b\right)=\frac{\left[\bar{V}-p_h^b\frac{\alpha}{2}V^{+}\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]}$$

then we have

$$V'\left(p_h^b\right)=\frac{\frac{\alpha}{2}\left[\bar{V}-V^{+}\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]^2}<0$$

$$\Rightarrow V\left(p_h^b\right)\leq V\left(0\right)=\bar{V}$$

Thus

$$\Rightarrow E\left(V_\omega\left|Q=-1\right.\right)=\frac{\left[\bar{V}-p_h^b\frac{\alpha}{2}V^{+}\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]}\leq\bar{V}$$

So we have the range of price when $Q=-1$ will be $P_{-1}\in\left[0,\bar{V}\right]$.

Next we discuss the sign of $\Delta_{\emptyset}^{n,s}=\frac{1}{3}\left[\pi_{-1}\bar{V}-P_{-1}-sP_{1}\right]$.

Case 1. If $p_{h}^{b}>\frac{2\bar{V}}{\alpha V^{+}}$, then by Lemma 2 we have $E\left(V_{\omega}\left|Q=-1\right.\right)<0$ and $P_{-1}=0$, so the firm will not invest when $Q=-1$, i.e. $\pi_{-1}=0$.

$$\Rightarrow \Delta_{\emptyset}^{n,s}=-\frac{1}{3}sP_{1}<0$$

Case 2. If $0<p_{h}^{b}\le\frac{2\bar{V}}{\alpha V^{+}}$, then we have

$$\Delta_{\emptyset}^{n,s}=\frac{1}{3}\left[\pi_{-1}\bar{V}-P_{-1}-sP_{1}\right]\le\frac{1}{3}\left[\bar{V}-P_{-1}-sP_{1}\right]$$

Let

$$\delta=\bar{V}-P_{-1}-sP_{1}$$

then we have

$$\Delta_{\emptyset}^{n,s}\le\frac{1}{3}\delta$$

Since $0<p_{h}^{b}\le\frac{2\bar{V}}{\alpha V^{+}}$, so the positively informed speculator is mixing between buying and doing nothing, which gives us

$$\Delta_{h}^{b,n}=\frac{1}{3}\left[3V_{h}-P_{0}-P_{1}-P_{2}\right]+\frac{s}{3}\left[P_{-1}-P_{2}\right]=0$$

Notice that from Lemma 2.1 $P_{0}=\bar{V}$, and the expected value of V_{ω} given $Q=2$ is

$$\begin{aligned}
E\left(V_{\omega}\left|Q=2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_{h}^{b}\frac{\alpha}{2}V^{+}+p_{l}^{b}\frac{\alpha}{2}V^{-}+p_{\emptyset}^{b}\left(1-\alpha\right)\bar{V}\right]\\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_{h}^{b}\frac{\alpha}{2}V^{+}\right]\\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}p_{h}^{b}\right]}\left[p_{h}^{b}\frac{\alpha}{2}V^{+}\right]=V^{+}
\end{aligned}$$

So we have $P_{2}=V^{+}$, thus

$$\Rightarrow \Delta_{h}^{b,n}=\frac{1}{3}\left[2V^{+}-\bar{V}-P_{1}\right]+\frac{s}{3}\left[P_{-1}-V^{+}\right]=0$$

$$\Rightarrow s=\frac{2V^{+}-\bar{V}-P_{1}}{V^{+}-P_{-1}}$$

Substitute s into δ gives us

$$\Rightarrow \delta = \bar{V} - P_{-1} - \frac{2V^{+} - \bar{V} - P_{1}}{V^{+} - P_{-1}} P_{1}$$

Then we have

$$\delta < 0 \Leftrightarrow \bar{V} - P_{-1} - \frac{2V^{+} - \bar{V} - P_{1}}{V^{+} - P_{-1}} P_{1} < 0$$

Rearrange the inequality yields

$$\delta < 0 \Leftrightarrow (P_{-1})^{2} - \left(V^{+} + \bar{V}\right) P_{-1} + V^{+}\bar{V} < -(P_{1})^{2} + \left(2V^{+} - \bar{V}\right) P_{1}$$

Let

$$f\left(P_{-1}\right) = (P_{-1})^{2} - \left(V^{+} + \bar{V}\right) P_{-1} + V^{+}\bar{V}$$

and

$$g\left(P_{1}\right) = -(P_{1})^{2} + \left(2V^{+} - \bar{V}\right) P_{1}$$

Then we have

$$f'\left(P_{-1}\right) = 2P_{-1} - \left(V^{+} + \bar{V}\right) < 0 \quad for \quad P_{-1} \in \left[0, \bar{V}\right]$$

$$\Rightarrow f\left(P_{-1}\right) \leq f\left(0\right) = V^{+}\bar{V}$$

Also we have

$$g'\left(P_{1}\right) = -2P_{1} + \left(2V^{+} - \bar{V}\right) = \left\{ \begin{array}{lll} > 0 & for & P_{-1} \in \left(\bar{V}, V^{+} - \frac{\bar{V}}{2}\right) \\ = 0 & for & P_{-1} = V^{+} - \frac{\bar{V}}{2} \\ < 0 & for & P_{-1} \in (V^{+} - \frac{\bar{V}}{2}, V^{+}] \end{array} \right\}$$

Thus

$$g\left(P_{1}\right) \geq \min\left\{g\left(\bar{V}\right), g\left(V^{+}\right)\right\}$$

Since we have

$$g\left(\bar{V}\right) = -\left(\bar{V}\right)^2 + \left(2V^+ - \bar{V}\right)\bar{V} = V^+\bar{V} + \left(V^+ - 2\bar{V}\right)\bar{V} > V^+\bar{V}$$

notice that this inequality holds due to

$$V^+ > V^+ + V^- = 2\bar{V}$$

And

$$g\left(V^+\right) = -\left(V^+\right)^2 + \left(2V^+ - \bar{V}\right)V^+ = \left(V^+\right)^2 - V^+\bar{V} > V^+\left(2\bar{V}\right) - V^+\bar{V} = V^+\bar{V}$$

Together we have

$$\Rightarrow g\left(P_1\right) \geq \min\left\{g\left(\bar{V}\right), g\left(V^+\right)\right\} > V^+\bar{V}$$

Which means

$$\Rightarrow f\left(P_{-1}\right) \leq V^+\bar{V} < g\left(P_1\right)$$

Thus we have

$$\left(P_{-1}\right)^2 - \left(V^+ + \bar{V}\right)P_{-1} + V^+\bar{V} < -\left(P_1\right)^2 + \left(2V^+ - \bar{V}\right)P_1$$

$$\Rightarrow \delta < 0$$

$$\Rightarrow \Delta_{\emptyset}^{n,s} \leq \frac{1}{3}\delta < 0$$

Case 3. If $p_h^b = 0$, then since also $p_h^s = 0$, we have $p_h^n = 1$.

Therefore the expected value of V_ω given $Q = -1$ will be

$$\begin{aligned} E\left(V_\omega \left|Q = -1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \left(1-p_l^b\right)\frac{\alpha}{2}V^- + \left(1-p_\emptyset^b\right)\left(1-\alpha\right)\bar{V}\right] \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\ &= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] = \bar{V} \end{aligned}$$

So we have $\pi_{-1} = 1$ and $P_{-1} = \bar{V}$.

$$\Rightarrow \Delta_{\emptyset}^{n,s} = \frac{1}{3}\left[\pi_{-1}\bar{V} - P_{-1} - sP_1\right] = -\frac{1}{3}sP_1 < 0$$

In sum, $\Delta_{\emptyset}^{n,s}$ is less than zero in all three cases, combine with the result in Claim 2.11 $\Delta_{\emptyset}^{b,n} < 0$, we have $\Delta_{\emptyset}^{b,s} = \Delta_{\emptyset}^{b,n} + \Delta_{\emptyset}^{n,s}$ is also less than zero, so the uninformed speculator will always choose to sell.

To sum up, when the initial short position $s > 0$ is revealed, we will have $p_{\emptyset}^{s} = 1$, the uninformed speculator will always choose to sell. $\square$

Proof of Claim 2.13

Proof. When the initial short position $s > 0$ is revealed, then from Lemma 2.1, 2.4, 2.5, we have $\pi_0 = \pi_1 = \pi_2 = 1$ and $P_0 = \bar{V}$.

Given that $p_l^s = p_{\emptyset}^s = 1$, then the expected value of V_ω given $Q = 1$ will be

$$\begin{aligned} E\left(V_\omega \left| Q=1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^+ + \left(1-p_l^s\right)\frac{\alpha}{2}V^- + \left(1-p_{\emptyset}^s\right)\left(1-\alpha\right)\bar{V}\right] \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=1\right)}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^+\right] \\ &= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-p_h^s\right)\frac{\alpha}{2}\right]}\left[\left(1-p_h^s\right)\frac{\alpha}{2}V^+\right] = V^+ \end{aligned}$$

Also the expected value of V_ω given $Q = 2$ is

$$\begin{aligned} E\left(V_\omega \left| Q=2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_h^b\frac{\alpha}{2}V^+ + p_l^b\frac{\alpha}{2}V^- + p_{\emptyset}^b\left(1-\alpha\right)\bar{V}\right] \\ &= \frac{1}{3}\frac{1}{\Pr\left(Q=2\right)}\left[p_h^b\frac{\alpha}{2}V^+\right] \\ &= \frac{1}{3}\frac{1}{\frac{1}{3}\left[p_h^b\frac{\alpha}{2}\right]}\left[p_h^b\frac{\alpha}{2}V^+\right] = V^+ \end{aligned}$$

Thus we have $P_1 = P_2 = V^+$.

The relative payoff from buying vs. doing nothing for positively informed speculator is

$$\begin{aligned} \Delta_h^{b,n} = U_h^b - U_h^n &= V_h E_D\left[\pi_{D+1}\right] - E_D\left[P_{D+1}\right] + s\left(E_D\left[P_D\right] - E_D\left[P_{D+1}\right]\right) \\ &= \left[V_h\sum_{d=-1}^{1}\frac{\pi_{d+1}}{3} - \sum_{d=-1}^{1}\frac{P_{d+1}}{3}\right] + s\left[\sum_{d=-1}^{1}\frac{P_d}{3} - \sum_{d=-1}^{1}\frac{P_{d+1}}{3}\right] \\ &= \frac{1}{3}\sum_{d=-1}^{1}\left[V_h\pi_{d+1} - P_{d+1}\right] + \frac{s}{3}\sum_{d=-1}^{1}\left[P_d - P_{d+1}\right] \\ &= \frac{1}{3}\left[V^+ - \bar{V} - s\left(V^+ - P_{-1}\right)\right] \end{aligned}$$

The relative payoff from doing nothing vs. selling for positively informed speculator is

$$\begin{aligned}\Delta_h^{n,s} &= U_h^n - U_h^s = V_h E_D\left[\pi_{D-1}\right] - E_D\left[P_{D-1}\right] + s\left(E_D\left[P_{D-1}\right] - E_D\left[P_D\right]\right)\\&= \left[V_h \sum_{d=-1}^{1} \frac{\pi_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_{d-1}}{3}\right] + s\left[\sum_{d=-1}^{1} \frac{P_{d-1}}{3} - \sum_{d=-1}^{1} \frac{P_d}{3}\right]\\&= \frac{1}{3}\sum_{d=-1}^{1}\left[V_h \pi_{d-1} - P_{d-1}\right] + \frac{s}{3}\sum_{d=-1}^{1}\left[P_{d-1} - P_d\right]\\&= \frac{1}{3}\left[\left(\pi_{-2} + \pi_{-1} + 1\right)V^+ - P_{-2} - P_{-1} - \bar{V} - s\left(V^+ - P_{-2}\right)\right]\end{aligned}$$

The relative payoff from buying vs. selling for positively informed speculator is

$$\begin{aligned}\Delta_h^{b,s} &= \Delta_h^{b,n} + \Delta_h^{n,s}\\&= \frac{1}{3}\left[\left(\pi_{-2} + \pi_{-1} + 2\right)V^+ - P_{-2} - P_{-1} - 2\bar{V} - s\left(2V^+ - P_{-2} - P_{-1}\right)\right]\end{aligned}$$

For the positively informed speculator, we can enumerate all different combinations for the sign of relative payoff in Table A2.6.3

Table A2.6.3: Summary of the Sign of Positively Informed Speculator's Relative Payoff

Relative payoff	Sign of Positively Informed Speculator's Relative Payoff								
$\Delta_h^{b,n}$	+	+	+	+	+	+	+	+	+
$\Delta_h^{n,s}$	+	+	+	0	0	0	−	−	−
$\Delta_h^{b,s}$	+	0	−	+	0	−	+	0	−
strategy	*b*	×	×	*b*	×	×	*b*	*bs*	*s*
$\Delta_h^{b,n}$	0	0	0	0	0	0	0	0	0
$\Delta_h^{n,s}$	+	+	+	0	0	0	−	−	−
$\Delta_h^{b,s}$	+	0	−	+	0	−	+	0	−
strategy	*bn*	×	×	×	*bns*	×	×	×	*s*
$\Delta_h^{b,n}$	−	−	−	−	−	−	−	−	−
$\Delta_h^{n,s}$	+	+	+	0	0	0	−	−	−
$\Delta_h^{b,s}$	+	0	−	+	0	−	+	0	−
strategy	*n*	*n*	*n*	×	×	*ns*	×	×	*s*

In the table above, notice that there will be 27 cases in total, however, since we have $\Delta_h^{b,s} = \Delta_h^{b,n} + \Delta_h^{n,s}$, so 14 cases are mathematically impossible, which have been marked as $\times$. For the rest 13 cases, we can reduce them to 7 cases according to the actions being chosen.

The reduced results are summarized in Table A2.6.4. So next we will discuss these 7 cases in detail.

Table A2.6.4: Reduced Summary of the Sign of Positively Informed Speculator's Relative Payoff

Relative payoff	Sign of Positively Informed Speculator's Relative Payoff						
$\Delta_h^{b,n}$	$+$	$+,0,-$	$-$	0	$-$	$+$	0
$\Delta_h^{n,s}$	$+,0,-$	$-$	$+$	$+$	0	$-$	0
$\Delta_h^{b,s}$	$+$	$-$	$+,0,-$	$+$	$-$	0	0
strategy	b	s	n	b,n	n,s	b,s	b,n,s

Case 1. If $\Delta_h^{b,n} > 0$, $\Delta_h^{b,s} > 0$, then we have $p_h^b = 1$, the positively informed speculator will always choose to buy.

$$\Delta_h^{b,n} > 0 \Rightarrow s < \frac{V^+ - \bar{V}}{V^+ - P_{-1}}$$

$$\Delta_h^{b,s} > 0 \Rightarrow s < \frac{(\pi_{-2} + \pi_{-1} + 2)\, V^+ - P_{-2} - P_{-1} - 2\bar{V}}{2V^+ - P_{-2} - P_{-1}}$$

Since $p_h^b = 1$, $p_h^s = 0$, $p_h^n = 0$, then by Lemma 2.2 and 2.3 we have, $\pi_{-2} = \pi_{-1} = 0$, $P_{-2} = P_{-1} = 0$, thus

$$\Rightarrow \left\{ \begin{array}{l} \Delta_h^{b,n} > 0 \Rightarrow s < 1 - \frac{\bar{V}}{V^+} \\ \\ \Delta_h^{b,s} > 0 \Rightarrow s < 1 - \frac{\bar{V}}{V^+} \end{array} \right\} \Rightarrow s < 1 - \frac{\bar{V}}{V^+}$$

Case 2. If $\Delta_h^{b,n} < 0$, $\Delta_h^{n,s} > 0$, then we have $p_h^n = 1$, the positively informed speculator will not choose to trade.

$$\Delta_h^{b,n} < 0 \Rightarrow s > \frac{V^+ - \bar{V}}{V^+ - P_{-1}}$$

$$\Delta_h^{n,s} > 0 \Rightarrow s < \frac{(\pi_{-2} + \pi_{-1} + 1) V^+ - P_{-2} - P_{-1} - \bar{V}}{V^+ - P_{-2}}$$

Since $p_h^n = 1$, $p_h^s = 0$, $p_h^b = 0$, then by Lemma 3 we have, $\pi_{-2} = 0$, $P_{-2} = 0$. Notice that the expected value of V_ω given $Q = -1$ is

$$\begin{aligned} E(V_\omega | Q = -1) &= \frac{1}{3} \frac{1}{\Pr(Q = -1)} \left[\left(1 - p_h^b\right) \frac{\alpha}{2} V^+ + \left(1 - p_l^b\right) \frac{\alpha}{2} V^- + \left(1 - p_\emptyset^b\right) (1 - \alpha) \bar{V} \right] \\ &= \frac{1}{3} \frac{1}{\Pr(Q = -1)} \left[\frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- + (1 - \alpha) \bar{V} \right] \\ &= \frac{1}{3} \frac{1}{\frac{1}{3} \left[\frac{\alpha}{2} + \frac{\alpha}{2} + (1 - \alpha) \right]} \left[\frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- + (1 - \alpha) \bar{V} \right] = \bar{V} \end{aligned}$$

So we have $\pi_{-1} = 1$, and $P_{-1} = \bar{V}$, thus

$$\Rightarrow \left\{ \begin{array}{ll} \Delta_h^{b,n} < 0 \Rightarrow & s > 1 \\ & \\ \Delta_h^{n,s} > 0 \Rightarrow s < 2 - \frac{2\bar{V}}{V^+} & \end{array} \right\} \Rightarrow 1 < s < 2 - \frac{2\bar{V}}{V^+}$$

Case 3. If $\Delta_h^{n,s} < 0$, $\Delta_h^{b,s} < 0$, then we have $p_h^s = 1$, the positively informed speculator will always choose to sell.

$$\Delta_h^{b,s} < 0 \Rightarrow s > \frac{(\pi_{-2} + \pi_{-1} + 2) V^+ - P_{-2} - P_{-1} - 2\bar{V}}{2V^+ - P_{-2} - P_{-1}}$$

$$\Delta_h^{n,s} < 0 \Rightarrow s > \frac{(\pi_{-2} + \pi_{-1} + 1) V^+ - P_{-2} - P_{-1} - \bar{V}}{V^+ - P_{-2}}$$

Since $p_h^s = 1$, $p_h^b = 0$, $p_h^n = 0$, then the expected value of V_ω given $Q = -1$ is

$$\begin{aligned} E(V_\omega | Q = -1) &= \frac{1}{3} \frac{1}{\Pr(Q = -1)} \left[\left(1 - p_h^b\right) \frac{\alpha}{2} V^+ + \left(1 - p_l^b\right) \frac{\alpha}{2} V^- + \left(1 - p_\emptyset^b\right) (1 - \alpha) \bar{V} \right] \\ &= \frac{1}{3} \frac{1}{\Pr(Q = -1)} \left[\frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- + (1 - \alpha) \bar{V} \right] \\ &= \frac{1}{3} \frac{1}{\frac{1}{3} \left[\frac{\alpha}{2} + \frac{\alpha}{2} + (1 - \alpha) \right]} \left[\frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- + (1 - \alpha) \bar{V} \right] = \bar{V} \end{aligned}$$

Also the expected value of V_ω given $Q=-2$ is

$$\begin{aligned}
E\left(V_\omega \left| Q=-2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + p_l^s\frac{\alpha}{2}V^- + p_\emptyset^s\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] = \bar{V}
\end{aligned}$$

So we have $\pi_{-2}=\pi_{-1}=1$, and $P_{-2}=P_{-1}=\bar{V}$, thus

$$\Rightarrow \left\{\begin{array}{c} \Delta_h^{n,s}<0 \Rightarrow s>3 \\ \\ \Delta_h^{b,s}<0 \Rightarrow s>2 \end{array}\right\} \Rightarrow s>3$$

Case 4. If $\Delta_h^{b,n}=0$, $\Delta_h^{n,s}>0$, $\Delta_h^{b,s}>0$, then we have $p_h^s=0$ and $p_h^b+p_h^n=1$, the positively informed speculator will mix between buying and doing nothing.

$$\Delta_h^{b,n}=0 \Rightarrow s=\frac{V^+-\bar{V}}{V^+-P_{-1}}$$

$$\Delta_h^{n,s}>0 \Rightarrow s<\frac{\left(\pi_{-2}+\pi_{-1}+1\right)V^+-P_{-2}-P_{-1}-\bar{V}}{V^+-P_{-2}}$$

Since $p_h^s=0$, then by Lemma 2.3 we have, $\pi_{-2}=0$, $P_{-2}=0$. Notice that the expected value of V_ω given $Q=-1$ is

$$\begin{aligned}
E\left(V_\omega \left| Q=-1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \left(1-p_l^b\right)\frac{\alpha}{2}V^- + \left(1-p_\emptyset^b\right)\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-p_h^b\right)\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{\left[\bar{V}-p_h^b\frac{\alpha}{2}V^+\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]}
\end{aligned}$$

Let

$$V\left(p_h^b\right)=\frac{\left[\bar{V}-p_h^b\frac{\alpha}{2}V^+\right]}{\left[1-p_h^b\frac{\alpha}{2}\right]}$$

then we have

$$V'\left(p_h^b\right) = \frac{\frac{\alpha}{2}\left[\bar{V} - V^+\right]}{\left[1 - p_h^b \frac{\alpha}{2}\right]^2} < 0$$

$$\Rightarrow V\left(p_h^b\right) \leq V(0) = \bar{V}$$

Thus

$$\Rightarrow E\left(V_\omega \,|Q = -1\right) = \frac{\left[\bar{V} - p_h^b \frac{\alpha}{2} V^+\right]}{\left[1 - p_h^b \frac{\alpha}{2}\right]} \leq \bar{V}$$

So we have the range of price when $Q = -1$ will be $P_{-1} \in \left[0, \bar{V}\right]$.

$$\Delta_h^{b,n} = 0 \Rightarrow s = \frac{V^+ - \bar{V}}{V^+ - P_{-1}} \in \left[1 - \frac{\bar{V}}{V^+}, 1\right]$$

$$\Delta_h^{n,s} > 0 \Rightarrow s < \frac{\left(\pi_{-1} + 1\right) V^+ - P_{-1} - \bar{V}}{V^+}$$

Substitute s into the inequality, yields

$$\Rightarrow s < \pi_{-1} - \frac{\bar{V}}{V^+} + \frac{V^+ - \bar{V}}{sV^+}$$

Rearrange

$$\Rightarrow s^2 - \left(\pi_{-1} - \frac{\bar{V}}{V^+}\right) s - \left(1 - \frac{\bar{V}}{V^+}\right) < 0$$

When $0 < P_{-1} \leq \bar{V}$, then we have $\pi_{-1} = 1$, and $1 - \frac{\bar{V}}{V^+} < s \leq 1$, then inequality will be

$$\Rightarrow s^2 - \left(1 - \frac{\bar{V}}{V^+}\right) s - \left(1 - \frac{\bar{V}}{V^+}\right) < 0$$

$$\Rightarrow \underbrace{(s-1)\left(s + \frac{\bar{V}}{V^+}\right)}_{\leq 0} + \underbrace{\left(\frac{2\bar{V}}{V^+} - 1\right)}_{<0} < 0$$

Thus the inequality will always hold for $1 - \frac{\bar{V}}{V^+} < s \leq 1$, $0 < P_{-1} \leq \bar{V}$. As for $P_{-1} = 0$, we need $s = 1 - \frac{\bar{V}}{V^+}$, which is then non-generic in this case.

Case 5. If $\Delta_h^{b,n} < 0$, $\Delta_h^{n,s} = 0$, $\Delta_h^{b,s} < 0$, then we have $p_h^b = 0$ and $p_h^n + p_h^s = 1$, the positively informed speculator will mix between doing nothing and selling.

$$\Delta_h^{n,s} = 0 \Rightarrow s = \frac{\left(\pi_{-2} + \pi_{-1} + 1\right) V^+ - P_{-2} - P_{-1} - \bar{V}}{V^+ - P_{-2}}$$

$$\Delta_h^{b,n} < 0 \Rightarrow s > \frac{V^+ - \bar{V}}{V^+ - P_{-1}}$$

Since the expected value of V_ω given $Q = -1$ is

$$\begin{aligned}
E\left(V_\omega \left| Q = -1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \left(1-p_l^b\right)\frac{\alpha}{2}V^- + \left(1-p_\emptyset^b\right)\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] = \bar{V}
\end{aligned}$$

then we have $\pi_{-1} = 1$, and $P_{-1} = \bar{V}$.

Also the expected value of V_ω given $Q = -2$ is

$$\begin{aligned}
E\left(V_\omega \left| Q = -2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + p_l^s\frac{\alpha}{2}V^- + p_\emptyset^s\left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[p_h^s\frac{\alpha}{2}+\frac{\alpha}{2}+\left(1-\alpha\right)\right]}\left[p_h^s\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + \left(1-\alpha\right)\bar{V}\right] \\
&= \frac{\left[\bar{V} - \left(1-p_h^s\right)\frac{\alpha}{2}V^+\right]}{\left[1-\left(1-p_h^s\right)\frac{\alpha}{2}\right]} = \frac{\left[\bar{V} - p_h^n\frac{\alpha}{2}V^+\right]}{\left[1-p_h^n\frac{\alpha}{2}\right]}
\end{aligned}$$

Let

$$V\left(p_h^n\right) = \frac{\left[\bar{V} - p_h^n\frac{\alpha}{2}V^+\right]}{\left[1-p_h^n\frac{\alpha}{2}\right]}$$

then we have

$$V'\left(p_h^n\right) = \frac{\frac{\alpha}{2}\left[\bar{V} - V^+\right]}{\left[1-p_h^n\frac{\alpha}{2}\right]^2} < 0$$

$$\Rightarrow V\left(p_h^n\right) \leq V\left(0\right) = \bar{V}$$

Thus

$$\Rightarrow E\left(V_\omega \left| Q = -2\right.\right) = \frac{\left[\bar{V} - p_h^n\frac{\alpha}{2}V^+\right]}{\left[1-p_h^n\frac{\alpha}{2}\right]} \leq \bar{V}$$

So we have the range of price when $Q = -2$ will be $P_{-2} \in \left[0, \bar{V}\right]$.

$$\Delta_h^{n,s} = 0 \Rightarrow s = \frac{\left(\pi_{-2}+2\right)V^+ - P_{-2} - 2\bar{V}}{V^+ - P_{-2}} = 1 + \frac{\pi_{-2}V^+ + \left(V^+ - 2\bar{V}\right)}{V^+ - P_{-2}} > 1$$

$$\Delta_h^{b,n} < 0 \Rightarrow s > 1$$

Thus the second inequality will always hold in this case, so next we will solve for the range of s

When $0 < P_{-2} \leq \bar{V}$, then we have $\pi_{-2} = 1$

$$\Rightarrow s = \frac{3V^+ - P_{-2} - 2\bar{V}}{V^+ - P_{-2}} = 1 + \frac{2V^+ - 2\bar{V}}{V^+ - P_{-2}} \in (3 - \frac{2\bar{V}}{V^+}, 3]$$

When $0 < P_{-2} \leq \bar{V}$, then we have $\pi_{-2} \in [0, 1]$

$$\Rightarrow s = (\pi_{-2} + 2) - \frac{2\bar{V}}{V^+} \in \left[2 - \frac{2\bar{V}}{V^+}, 3 - \frac{2\bar{V}}{V^+}\right]$$

Together we have, $2 - \frac{2\bar{V}}{V^+} \leq s \leq 3$.

Notice that in order to make the firm mixing between invest and not invest, we need the expected value of V_ω given $Q = -2$ to be zero, which means

$$E\left(V_\omega \,|Q = -2\right) = \frac{\left[\bar{V} - \left(1 - p_h^s\right)\frac{\alpha}{2}V^+\right]}{\left[1 - \left(1 - p_h^s\right)\frac{\alpha}{2}\right]} = \frac{\left[\bar{V} - p_h^n\frac{\alpha}{2}V^+\right]}{\left[1 - p_h^n\frac{\alpha}{2}\right]} = 0$$

$$\Rightarrow p_h^s = 1 - \frac{2\bar{V}}{\alpha V^+}, \quad p_h^n = \frac{2\bar{V}}{\alpha V^+}$$

Case 6. If $\Delta_h^{b,n} > 0$, $\Delta_h^{n,s} < 0$, $\Delta_h^{b,s} = 0$, then we have $p_h^n = 0$ and $p_h^b + p_h^s = 1$, the positively informed speculator will mix between buying and selling.

$$\Delta_h^{b,s} = 0 \Rightarrow s = \frac{\left(\pi_{-2} + \pi_{-1} + 2\right)V^+ - P_{-2} - P_{-1} - 2\bar{V}}{2V^+ - P_{-2} - P_{-1}}$$

$$\Delta_h^{b,n} > 0 \Rightarrow s < \frac{V^+ - \bar{V}}{V^+ - P_{-1}}$$

First, when $p_h^b < \frac{2\bar{V}}{\alpha V^+}$, since $p_h^n = 0$, then $p_h^s < 1 - \frac{2\bar{V}}{\alpha V^+}$. By Lemma 2.2 and 2.3, we will have $\pi_{-2} = \pi_{-1} = 0$ and $P_{-2} = P_{-1} = 0$.

$$\Rightarrow \left\{ \begin{array}{l} \Delta_h^{b,s} = 0 \Rightarrow s = 1 - \frac{\bar{V}}{V^+} \\ \\ \Delta_h^{b,n} > 0 \Rightarrow s < 1 - \frac{\bar{V}}{V^+} \end{array} \right\} \Rightarrow s \in \emptyset$$

Second, when $p_h^b = \frac{2\bar{V}}{\alpha V^+}$, since $p_h^n = 0$, then $p_h^s = 1 - \frac{2\bar{V}}{\alpha V^+}$.

The expected value of V_ω given $Q = -1$ will be

$$\begin{aligned}
E\left(V_\omega \left| Q = -1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \left(1-p_l^b\right)\frac{\alpha}{2}V^- + \left(1-p_\emptyset^b\right)(1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-\frac{2\bar{V}}{\alpha V^+}\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] = 0
\end{aligned}$$

The expected value of V_ω given $Q = -2$ will be

$$\begin{aligned}
E\left(V_\omega \left| Q = -2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + p_l^s\frac{\alpha}{2}V^- + p_\emptyset^s(1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[\left(1-\frac{2\bar{V}}{\alpha V^+}\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] = 0
\end{aligned}$$

Thus we have, $P_{-1} = P_{-2} = 0$.

$$\Rightarrow \left\{\begin{array}{c}
\Delta_h^{b,s} = 0 \Rightarrow s = \frac{(\pi_{-2}+\pi_{-1}+2)V^+ - 2\bar{V}}{2V^+} \geq \frac{2V^+ - 2\bar{V}}{2V^+} = 1 - \frac{\bar{V}}{V^+} \\
\\
\Delta_h^{b,n} > 0 \Rightarrow s < 1 - \frac{\bar{V}}{V^+}
\end{array}\right\} \Rightarrow s \in \emptyset$$

Finally, when $p_h^b < \frac{2\bar{V}}{\alpha V^+}$, since $p_h^n = 0$, then $p_h^s > 1 - \frac{2\bar{V}}{\alpha V^+}$.

The expected value of V_ω given $Q = -1$ will be

$$\begin{aligned}
E\left(V_\omega \left| Q = -1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \left(1-p_l^b\right)\frac{\alpha}{2}V^- + \left(1-p_\emptyset^b\right)(1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-p_h^b\right)\frac{\alpha}{2} + \frac{\alpha}{2} + (1-\alpha)\right]}\left[\left(1-p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{\left[\bar{V} - p_h^b\frac{\alpha}{2}V^+\right]}{\left[1 - p_h^b\frac{\alpha}{2}\right]}
\end{aligned}$$

Let

$$V\left(p_h^b\right) = \frac{\left[\bar{V} - p_h^b\frac{\alpha}{2}V^+\right]}{\left[1 - p_h^b\frac{\alpha}{2}\right]}$$

then we have

$$V'\left(p_h^b\right) = \frac{\frac{\alpha}{2}\left[\bar{V} - V^+\right]}{\left[1 - p_h^b\frac{\alpha}{2}\right]^2} < 0$$

$$\Rightarrow V\left(\frac{2\bar{V}}{\alpha V^{+}}\right) < V\left(p_h^b\right) \leq V(0)$$
$$\Rightarrow 0 < V\left(p_h^b\right) \leq \bar{V}$$

Thus

$$\Rightarrow 0 < E\left(V_\omega \left| Q = -1\right.\right) \leq \bar{V}$$

The expected value of V_ω given $Q = -2$ will be

$$\begin{aligned}
E\left(V_\omega \left| Q = -2\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q = -2\right)}\left[p_h^s \frac{\alpha}{2} V^{+} + p_l^s \frac{\alpha}{2} V^{-} + p_\emptyset^s \left(1 - \alpha\right) \bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q = -2\right)}\left[p_h^s \frac{\alpha}{2} V^{+} + \frac{\alpha}{2} V^{-} + \left(1 - \alpha\right) \bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[p_h^s \frac{\alpha}{2} + \frac{\alpha}{2} + (1-\alpha)\right]}\left[p_h^s \frac{\alpha}{2} V^{+} + \frac{\alpha}{2} V^{-} + \left(1 - \alpha\right) \bar{V}\right] \\
&= \frac{\left[\bar{V} - \left(1 - p_h^s\right)\frac{\alpha}{2} V^{+}\right]}{\left[1 - \left(1 - p_h^s\right)\frac{\alpha}{2}\right]} = \frac{\left[\bar{V} - p_h^b \frac{\alpha}{2} V^{+}\right]}{\left[1 - p_h^b \frac{\alpha}{2}\right]} = E\left(V_\omega \left| Q = -1\right.\right)
\end{aligned}$$

Thus

$$\Rightarrow 0 < E\left(V_\omega \left| Q = -2\right.\right) \leq \bar{V}$$

So we have, $\pi_{-2} = \pi_{-1} = 1$, $P_{-2} \in (0, \bar{V}]$ and $P_{-1} \in (0, \bar{V}]$.

$$\Rightarrow \left\{ \begin{array}{c} \Delta_h^{b,s} = 0 \Rightarrow s = \frac{4V^{+} - P_{-1} - P_{-2} - 2\bar{V}}{2V^{+} - P_{-1} - P_{-2}} = 1 + \frac{2V^{+} - 2\bar{V}}{2V^{+} - P_{-1} - P_{-2}} > 1 \\ \\ \Delta_h^{b,n} > 0 \Rightarrow s < \frac{V^{+} - \bar{V}}{V^{+} - P_{-1}} \leq 1 \end{array} \right\} \Rightarrow s \in \emptyset$$

In sum, there is no such s exists to make positively informed speculator mix between buying and selling.

Case 7. If $\Delta_h^{b,n} = 0$, $\Delta_h^{n,s} = 0$, $\Delta_h^{b,s} = 0$, then we have $p_h^b + p_h^n + p_h^s = 1$, the positively informed speculator will mix between buying, selling and doing nothing.

$$\Delta_h^{b,n} = 0 \Rightarrow s = \frac{V^{+} - \bar{V}}{V^{+} - P_{-1}}$$
$$\Delta_h^{n,s} = 0 \Rightarrow s = \frac{\left(\pi_{-2} + \pi_{-1} + 1\right) V^{+} - P_{-2} - P_{-1} - \bar{V}}{V^{+} - P_{-2}}$$

Let

$$s_1 = \frac{V^+ - \bar{V}}{V^+ - P_{-1}}$$

$$s_2 = \frac{(\pi_{-2} + \pi_{-1} + 1)\, V^+ - P_{-2} - P_{-1} - \bar{V}}{V^+ - P_{-2}}$$

First, when $p_h^b < \frac{2\bar{V}}{\alpha V^+}$, then the expected value of V_ω given $Q = -1$ is

$$\begin{aligned}
E\left(V_\omega \,|Q = -1\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q = -1\right)}\left[\left(1 - p_h^b\right)\frac{\alpha}{2}V^+ + \left(1 - p_l^b\right)\frac{\alpha}{2}V^- + \left(1 - p_\emptyset^b\right)(1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q = -1\right)}\left[\left(1 - p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1 - p_h^b\right)\frac{\alpha}{2} + \frac{\alpha}{2} + (1-\alpha)\right]}\left[\left(1 - p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{\left[\bar{V} - p_h^b\frac{\alpha}{2}V^+\right]}{\left[1 - p_h^b\frac{\alpha}{2}\right]} \in (0, \bar{V}]
\end{aligned}$$

The expected value of V_ω given $Q = -2$ is

$$\begin{aligned}
E\left(V_\omega \,|Q = -2\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q = -2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + p_l^s\frac{\alpha}{2}V^- + p_\emptyset^s\,(1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q = -2\right)}\left[p_h^s\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[p_h^s\frac{\alpha}{2} + \frac{\alpha}{2} + (1-\alpha)\right]}\left[p_h^s\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{\left[\bar{V} - (1 - p_h^s)\,\frac{\alpha}{2}V^+\right]}{\left[1 - \left(1 - p_h^s\right)\frac{\alpha}{2}\right]} \in \left[0, \bar{V}\right]
\end{aligned}$$

Thus we have $\pi_{-1} = 1$, $P_{-1} \in (0, \bar{V}]$, $P_{-2} \in \left[0, \bar{V}\right]$.

$$\begin{aligned}
\Rightarrow s_2 &= \frac{(\pi_{-2} + 2)\, V^+ - P_{-2} - P_{-1} - \bar{V}}{V^+ - P_{-2}} \\
&\geq \frac{2V^+ - P_{-2} - P_{-1} - \bar{V}}{V^+ - P_{-2}} \\
&= 1 + \frac{V^+ - P_{-1} - \bar{V}}{V^+ - P_{-2}} \\
&\geq 1 + \frac{V^+ - P_{-1} - \bar{V}}{V^+}
\end{aligned}$$

Since

$$s_1 = \frac{V^+ - \bar{V}}{V^+ - P_{-1}} < 1 + \frac{V^+ - P_{-1} - \bar{V}}{V^+} \Leftrightarrow 0 < \left(P_{-1}\right)^2 - \left(3V^+ - \bar{V}\right)P_{-1} + \left(V^+\right)^2$$

Let

$$\theta\left(P_{-1}\right)=\left(P_{-1}\right)^{2}-\left(3V^{+}-\bar{V}\right)P_{-1}+\left(V^{+}\right)^{2}$$

then we have

$$\theta'\left(P_{-1}\right)=2P_{-1}+\bar{V}-3V^{+}<0 \quad for \quad P_{-1}\in(0,\bar{V}]$$

$$\Rightarrow \theta\left(P_{-1}\right)\geq\theta\left(\bar{V}\right)=\left(V^{+}\right)^{2}-3V^{+}\bar{V}+2\left(\bar{V}\right)^{2}=\left(V^{+}-\bar{V}\right)\left(V^{+}-2\bar{V}\right)>0$$

Thus for all $P_{-1}\in(0,\bar{V}]$, there will be

$$\left(P_{-1}\right)^{2}-\left(3V^{+}-\bar{V}\right)P_{-1}+\left(V^{+}\right)^{2}>0$$

$$\Rightarrow s_{1}<1+\frac{V^{+}-P_{-1}-\bar{V}}{V^{+}}$$

Since $s_1<1+\frac{V^{+}-P_{-1}-\bar{V}}{V^{+}}\leq s_2$, then we have $s\in\emptyset$.

Second, when $p_h^b\geq\frac{2\bar{V}}{\alpha V^{+}}$, then $p_h^s\leq 1-\frac{2\bar{V}}{\alpha V^{+}}$. The expected value of V_ω given $Q=-1$ will be

$$\begin{aligned}
E\left(V_{\omega}\left|Q=-1\right.\right)&=\frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_{h}^{b}\right)\frac{\alpha}{2}V^{+}+\left(1-p_{l}^{b}\right)\frac{\alpha}{2}V^{-}+\left(1-p_{\emptyset}^{b}\right)(1-\alpha)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\Pr\left(Q=-1\right)}\left[\left(1-p_{h}^{b}\right)\frac{\alpha}{2}V^{+}+\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1-p_{h}^{b}\right)\frac{\alpha}{2}+\frac{\alpha}{2}+(1-\alpha)\right]}\left[\left(1-p_{h}^{b}\right)\frac{\alpha}{2}V^{+}+\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\right]\\
&=\frac{\left[\bar{V}-p_{h}^{b}\frac{\alpha}{2}V^{+}\right]}{\left[1-p_{h}^{b}\frac{\alpha}{2}\right]}\leq\frac{\left[\bar{V}-\left(\frac{2\bar{V}}{\alpha V^{+}}\right)\frac{\alpha}{2}V^{+}\right]}{\left[1-p_{h}^{b}\frac{\alpha}{2}\right]}=0
\end{aligned}$$

And the expected value of V_ω given $Q=-2$ will be

$$\begin{aligned}
E\left(V_{\omega}\left|Q=-2\right.\right)&=\frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_{h}^{s}\frac{\alpha}{2}V^{+}+p_{l}^{s}\frac{\alpha}{2}V^{-}+p_{\emptyset}^{s}(1-\alpha)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\Pr\left(Q=-2\right)}\left[p_{h}^{s}\frac{\alpha}{2}V^{+}+\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\right]\\
&=\frac{1}{3}\frac{1}{\frac{1}{3}\left[p_{h}^{s}\frac{\alpha}{2}+\frac{\alpha}{2}+(1-\alpha)\right]}\left[p_{h}^{s}\frac{\alpha}{2}V^{+}+\frac{\alpha}{2}V^{-}+(1-\alpha)\bar{V}\right]\\
&=\frac{\left[\bar{V}-\left(1-p_{h}^{s}\right)\frac{\alpha}{2}V^{+}\right]}{\left[1-\left(1-p_{h}^{s}\right)\frac{\alpha}{2}\right]}\leq\frac{\left[\bar{V}-\left(\frac{2\bar{V}}{\alpha V^{+}}\right)\frac{\alpha}{2}V^{+}\right]}{\left[1-\left(1-p_{h}^{s}\right)\frac{\alpha}{2}\right]}=0
\end{aligned}$$

Thus we have $P_{-1} = P_{-2} = 0$.

$$\Rightarrow s_1 = \frac{V^+ - \bar{V}}{V^+ - P_{-1}} = 1 - \frac{\bar{V}}{V^+}$$
$$\Rightarrow s_2 = \frac{(\pi_{-2} + \pi_{-1} + 1)\, V^+ - \bar{V}}{V^+}$$

Then let $s_1 = s_2$, gives us

$$s_1 = s_2 \Rightarrow \frac{(\pi_{-2} + \pi_{-1} + 1)\, V^+ - \bar{V}}{V^+} = 1 - \frac{\bar{V}}{V^+}$$

$$\Rightarrow (\pi_{-2} + \pi_{-1})\, V^+ = 0$$

$$\Rightarrow \pi_{-2} = \pi_{-1} = 0$$

Therefore

$$s = 1 - \frac{\bar{V}}{V^+}$$

which is non-generic in this case.

In sum of all these 7 cases, we will have the trading strategies for the positively informed speculator as follow:

- If $s < 1 - \frac{\bar{V}}{V^+}$, then $p_h^b = 1$, the positively informed speculator will always choose to buy.
- If $1 - \frac{\bar{V}}{V^+} < s \leq 1$, then $p_h^b + p_h^n = 1$, the positively informed speculator will mix between buying and doing nothing.
- If $1 < s < 2 - \frac{2\bar{V}}{V^+}$, then $p_h^n = 1$, the positively informed speculator will always choose not to trade.
- If $2 - \frac{2\bar{V}}{V^+} \leq s \leq 3$, then $p_h^n + p_h^s = 1$, the positively informed speculator will mix between doing nothing and selling.
- If $3 < s$, then $p_h^s = 1$, the positively informed speculator will always choose to sell.

$\square$

Proof of Claim 2.14

Proof. When the initial short position $s > 0$ is revealed, the speculator's strategies are given by Claim 2.10-2.13, so according to Lemma 2.1-2.5, we have the firm's investment strategies will then be implied as:

- If $s < 1 - \frac{\bar{V}}{V^+}$, then firm will invest when $Q = 0, 1, 2$, and will not invest when $Q = -1, -2$.
- If $1 - \frac{\bar{V}}{V^+} < s < 2 - \frac{2\bar{V}}{V^+}$, then firm will invest when $Q = -1, 0, 1, 2$, and will not invest when $Q = -2$.
- If $2 - \frac{2\bar{V}}{V^+} \leq s$, then firm will always choose to invest, i.e. invest when $Q = -2, -1, 0, 1, 2$.

As for the pricing strategies of the market maker, if $s < 1 - \frac{\bar{V}}{V^+}$, then the firm chooses not to invest when $Q = -1, -2$, thus we have $P_{-1}{=}P_{-2}{=}0$. Since the positively informed speculator is the only type who will choose to buy, so we have $P_1{=}P_2{=}V^+$. Finally, we have $P_0{=}\bar{V}$ by Lemma 2.1.

If $1 - \frac{\bar{V}}{V^+} < s < 2 - \frac{2\bar{V}}{V^+}$, then firm will choose not to invest when $Q = -2$, thus we have $P_{-2}{=}0$. Since the positively informed speculator is the only type who will choose to buy, so we have $P_1{=}P_2{=}V^+$. Notice that the expected value of V_ω given $Q = -1$ is

$$
\begin{aligned}
E\left(V_\omega \left| Q = -1\right.\right) &= \frac{1}{3}\frac{1}{\Pr\left(Q = -1\right)}\left[\left(1 - p_h^b\right)\frac{\alpha}{2}V^+ + \left(1 - p_l^b\right)\frac{\alpha}{2}V^- + \left(1 - p_\emptyset^b\right)(1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\Pr\left(Q = -1\right)}\left[\left(1 - p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{1}{3}\frac{1}{\frac{1}{3}\left[\left(1 - p_h^b\right)\frac{\alpha}{2} + \frac{\alpha}{2} + (1-\alpha)\right]}\left[\left(1 - p_h^b\right)\frac{\alpha}{2}V^+ + \frac{\alpha}{2}V^- + (1-\alpha)\,\bar{V}\right] \\
&= \frac{\left[\bar{V} - p_h^b\frac{\alpha}{2}V^+\right]}{\left[1 - p_h^b\frac{\alpha}{2}\right]}
\end{aligned}
$$

So we have $P_{-1} = \frac{\left[\bar{V} - p_h^b\frac{\alpha}{2}V^+\right]}{\left[1 - p_h^b\frac{\alpha}{2}\right]}$. Finally, we have $P_0{=}\bar{V}$ by Lemma 2.1.

If $2 - \frac{2\bar{V}}{V^+} \leq s$, then firm will always choose to invest, i.e. invest when $Q = -2, -1, 0, 1, 2$. In this case, D1 implies $P_1{=}P_2{=}V^+$. And since $Q = -1, 0$ conveys no information, the price should be $P_{-1} = P_0 = \bar{V}$. Finally, the expected value of V_ω given $Q = -2$ is given by

$$\begin{aligned} E\left(V_\omega \left| Q = -2\right.\right) &= \frac{1}{3} \frac{1}{\Pr\left(Q = -2\right)} \left[p_h^s \frac{\alpha}{2} V^+ + p_l^s \frac{\alpha}{2} V^- + p_\emptyset^s \left(1 - \alpha\right) \bar{V}\right] \\ &= \frac{1}{3} \frac{1}{\Pr\left(Q = -2\right)} \left[p_h^s \frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- + \left(1 - \alpha\right) \bar{V}\right] \\ &= \frac{1}{3} \frac{1}{\frac{1}{3}\left[p_h^s \frac{\alpha}{2} + \frac{\alpha}{2} + \left(1 - \alpha\right)\right]} \left[p_h^s \frac{\alpha}{2} V^+ + \frac{\alpha}{2} V^- + \left(1 - \alpha\right) \bar{V}\right] \\ &= \frac{\left[\bar{V} - \left(1 - p_h^s\right) \frac{\alpha}{2} V^+\right]}{\left[1 - \left(1 - p_h^s\right) \frac{\alpha}{2}\right]} \end{aligned}$$

Thus we have $P_{-2} = \frac{\left[\bar{V} - \left(1 - p_h^s\right) \frac{\alpha}{2} V^+\right]}{\left[1 - \left(1 - p_h^s\right) \frac{\alpha}{2}\right]}$.

Below is the summary of the market maker's pricing strategies:

- If $s < 1 - \frac{\bar{V}}{V^+}$, then market maker will set the prices as

$$P_1 = P_2 = V^+$$

$$P_0 = \bar{V}$$

$$P_{-1} = P_{-2} = 0$$

- If $s < 1 - \frac{\bar{V}}{V^+}$, then market maker will set the prices as

$$P_1 = P_2 = V^+$$

$$P_0 = \bar{V}$$

$$P_{-1} = \frac{\left[\bar{V} - p_h^b \frac{\alpha}{2} V^+\right]}{\left[1 - p_h^b \frac{\alpha}{2}\right]}$$

$$P_{-2} = 0$$

- If $s < 1 - \frac{\bar{V}}{V^+}$, then market maker will set the prices as

$$P_1 = P_2 = V^+$$

$$P_0 = P_{-1} = \bar{V}$$

$$P_{-2} = \frac{\left[\bar{V} - \left(1 - p_h^s\right) \frac{\alpha}{2} V^+\right]}{\left[1 - \left(1 - p_h^s\right) \frac{\alpha}{2}\right]}$$

□

CHAPTER 3. UNDERSUBSCRIPTION RISK, UNDERPRICING, AND UNDERREACTION IN INITIAL PUBLIC OFFERINGS

Using theoretical and simulation tools, we study how strategic risk among investors can help explain both underpricing and underreaction in initial public offerings (IPOs). We assume the post-IPO value of a firm is higher if the IPO raises more capital for the firm. Hence an IPO subscriber faces strategic risk: the value of subscribing depends on the aggregate subscription rate. As this risk is resolved immediately after the IPO, the IPO itself is underpriced. Moreover, since individual investors have limited wealth, a higher offer price raises the risk of undersubscription. Investors respond by demanding a larger discount: the offer price appears to underreact to public news.

3.1 Introduction

Initial public offering (IPO) is an important milestone for entrepreneurial firms. The proceeds from IPO can help to finance the future growth of firms and make them more liquid through stock market trading. IPO also provides a way for trading the company's shares, enabling its existing shareholders to diversify their investments and to crystallize their capital gains from backing the company. The act of IPO itself helps improve the reputation of the company, and the attendant publicity may bring indirect benefits, such as attracting more talented managers and lowering the cost of funding the company's operations and investments.

An important aspect of the IPO process is the underpricing of newly issued shares, representing a discount from its fair market price measured by the difference between the closing price on the first day of trading and the IPO offer price. IPO underpricing is one of the best-documented empirical findings in finance and the underpricing phenomenon is persistent over time and across countries. Logue (1973) and Ibbotson (1975) documented that when companies go public, the shares they sell tend to be underpriced, in that the share price jumps substantially on the first day of trading. Ljungqvist (2004) provides evidence of underpricing in United States from 1960 to 2003, in main countries of Europe from 1990 to 2003 and in main countries of Asia-Pacific and Latin America from 1990 to 2001. To be specific, in United States,there are $8,249$ IPOs from 1980 to 2016. The average first-day return for the IPOs is 17.9% (equally-weighted average return) and the aggregate amount of "money left on the table" is $155.14 billion, where the "money left on the table" is defined as the first-day price gain multiplied by the number of shares sold. Compared with the total proceeds of IPO ($839.65 billion), 18.5% of firms' potential proceeds has been left to the investors. Figure 3.1 displays the mean first-day return and "money left on the table" for IPOs in United States from 1980-2016. Such "money left on the table" constitutes a substantial opportunity cost of going public for issuing firms. However,owners and managers seem unconcerned about situations of underpricing. In a survey of chief financial officers (CFOs) that took their firms public, Krigman et al. (2001) find that CFOs of virtually all of the most underpriced firms are highly satisfied with the performance of their lead IPO underwriter.

Why are the firms willing to sacrifice such great amount of money in the process of IPO? In this paper, we present an explanation for this underpricing phenomenon by examining the strategic risk in IPO. The concept of strategic risk comes from the global games literature. Generally, we consider a situation in which payoffs from agents strategies depend on an uncertain state of the world about which agents obtain very informative but noisy signals. Because agents do not have the same assessments of the state of the world, this creates strategic uncertainty in equilibrium. The risk comes from this kind of uncertainty is called strategic risk. Strategic risk is widely examined in researches on global games[1].

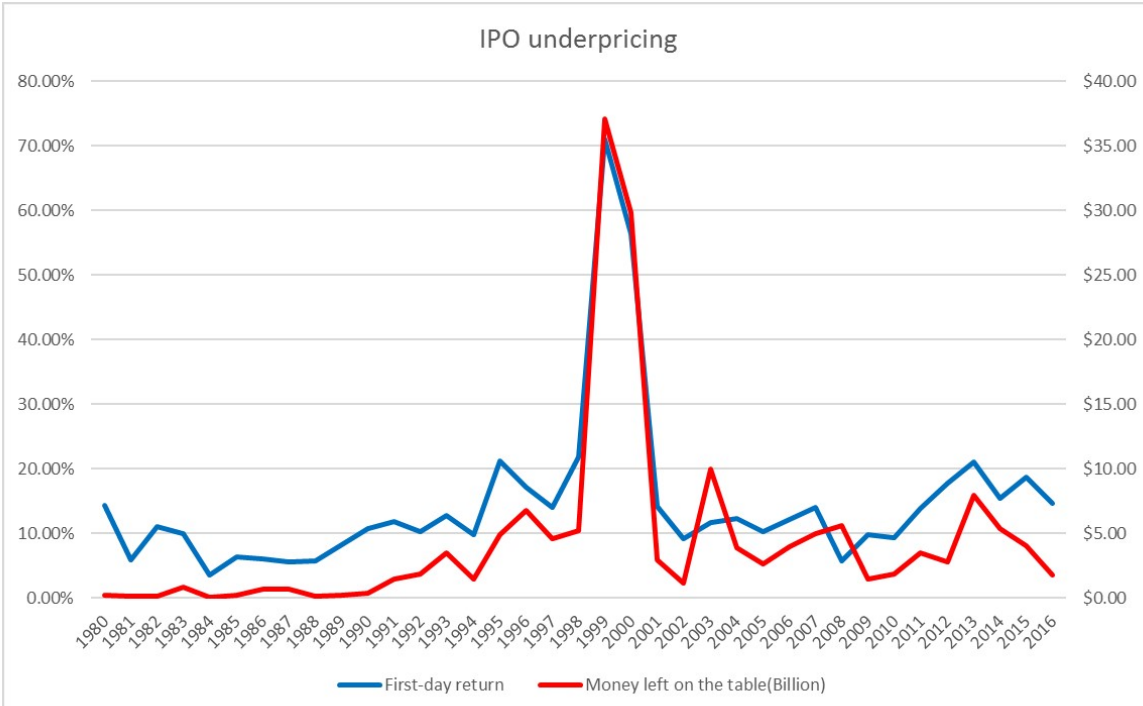

Figure 3.1: IPO Underpricing in US

However, strategic risk is seldom examined in literature on IPO. In our setting, shares sold in an IPO are more valuable if the firm reaps more revenue from the IPO. There are two motivations for this assumption. Trivially, a firm can use its IPO proceeds productively which lets it pay higher dividends in the future. In addition, Stoughton, Wong, and Zechner (2001) suggest that the success of a firm's IPO acts as a quality signal to the firm's small stakeholders, who may choose whether or not to do business with the firm or to adopt the firm's platform. Such stakeholders may include customers, suppliers, makers of ancillary products such as software and replacement parts, and potential future investors. For the firm to survive and thrive, such stakeholders must be willing to do business with it. Importantly, this informational effect may be large even if a firm seeks relatively little capital in its IPO. With the assumption that liquidating dividend is increasing in IPO proceeds, investors in

[1] For example, see Carlsson and van Damme (1993), Morris and Shin(1998), Chassang and Miquel(2010)

an IPO face strategic risk: the value of the shares depends on the IPO's success, which depends on other investors' decisions to subscribe.

We will show in a global games setting that the strategic risk leads to underpricing. Intuitively, an agent's reservation price is the price at which she is just willing to subscribe to the IPO. But if, given her information about the IPO, she is indeed just willing to subscribe, then she knows that some others are likely to have received slightly more negative information than her own and thus will choose not to subscribe. Hence, her reservation price reflects a positive probability of undersubscription, which - if it occurs - will lower the firm's value. Once the IPO concludes and the subscription rate is known, this strategic risk disappears. Accordingly, shares trade at a higher price in the aftermarket.[2]

Strategic risk can generate underreaction as well. The term underreaction describes the well-documented fact that the final offer price does not fully react to favorable information received in the process of pricing IPO, which indicates that the price revision over the course of bookbuilding and the first-day underpricing return are positively correlated[3]. Intuitively, when good news tells the firm that the IPO is more attractive to investors, the firm can raise the IPO price. But since individual investors have limited wealth, a higher offer price raises the risk of undersubscription. Hence the risk of undersubscription is now greater: investors face even more strategic risk. Thus, this price revision worsens underpricing, which appears in the data as underreaction.

Besides, our empirical result shows that underpricing is positively correlated with oversubscription, our model can also give an explanation for this. Intuitively, since firm sees only a noisy signal of fundamentals. Being concerned with the risk of undersubscription, when the firm observes a bad signal and is overly pessimistic, it will lower its offer price to attract more investor to subscribe. Therefore, there is more underpricing and more

[2]Underpricing in this model does not require risk aversion. Rather, it occurs because the firm must charge a low share price in order to induce the agents to subscribe in the presence of undersubscription risk. Indeed, our model assumes risk-neutrality; under risk aversion, the underpricing would be worse.

[3]For exmaple, see Hanley (1993), Edelen and Kadlec (2005),Bradley and Jordan (2002),Lowry and Schwert (2004)

oversubscription.

Historically, there are four main kinds of explanations for IPO underpricing. The first kind of explanation for underpricing claims that it is due to winner's curse [4] : when the firm's growth prospects are high, informed investors will subscribe, shrinking the stock available to retail investors. Since retail investors face a winner's curse, they are not willing to pay the firm the true value of its shares. This kind of explanation shows how underpricing happens. However, it did not explain for the underraction phenomenon and the positively relationship between oversubscription and underpricing. The second explanation of underpricing is the signal of firm quality[5]. If companies have better information about the present value or risk of their future cash flows than do investors, underpricing may be used to signal the companys "true" high value. This is clearly costly, but if successful, signaling may allow the issuer to return to the market to sell equity on better prices at a later date. This explanation does not explain for the underreaction phenomenon and the relationship between oversubscription and underpricing, either.

The third explanation for underpricing is moral hazard[6]. Intuitively, a firm conducts an IPO through a third party underwriter. The underwriting has an incentive to reward itself or top clients with underpriced shares. This theory can also be used to explain underreaction in IPO. However, this theory does not take the subscription rate into account. The fourth explanation is information revelation theories. Benveniste and Spindt (1989), Benveniste and Wilhelm (1990), and Spatt and Srivastava (1991) show that if some investors are better informed than either the company or other investors, underwriter has the incentive to design a mechanism through the process of bookbuilding which will induce investors to reveal their information truthfully by making it in their best interest to do so. To ensure truth-telling, the allocations have to involve underpriced stock. In this explanation, IPO underpricing serves as the cost of extracting the informed investors private information. Bookbuilding

[4]For example, see Rock (1986)
[5]For example, see Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989)
[6]See Baron and Holmstrm (1980), and Baron (1982)

allows firms to extract positive information and raise the offer price in response even though the price will rise further in the after-market because some money has to be left on the table. Thus the price revision over the course of bookbuilding and the first-day underpricing return are positively correlated. This setting can also explain the phenomenon of IPO underreaction. But it does not take the subscription into account, either.

This paper differs from the related literature in the following ways. First, unlike the theory of winner's curse and signaling, our model can explain the phenomenon of underpricing and underreaction at the same time. Although the theory of moral hazard and information revelation can also explain the underreaction, they did not take the subscription into account, which ignores the relationship between underpricing and oversubscription. Our theory can explain all these three phenomena. Second, different from the explanation of winner's curse, information revealing and signaling, there is no need for us to assume that there is a information gap (some investors are informed and some are uninformed) among investors. In fact, we can show that even when the investors share the same information, underpricing may still exist in our setting. Third, unlike the explanations which referred to moral hazard and psychological reasons, our paper assumes that all agents engaged in the IPO process are fully rational. With the assumption that moral hazard caused the underpricing, Baron and Holmstrm (1980), Baron (1982) construct a screening model where the uninformed party offers a menu, from which the informed party selects the one that is optimal given her unobserved type in the road show process. However, this kind of road show commitment is not widely observed in reality. Our paper gets rid of this commitment and tends to be more realistic. Fourth, our paper takes the endogeneity of stock value into consideration, which has seldom been examined before in the studies of IPO. IPO revenue can be used to finance firm's investment and a successful IPO will help to improve firm's reputation. So firm's value, and hence the stock value, may be affected by the IPO process itself. However, few research on IPO has examined on this effect. Our paper seeks to fill this gap.

3.2 The Model

There is a fixed measure $m > 0$ of agents, each endowed with one unit of capital. There is also a single firm with a worthwhile project. All participants are risk-neutral and fully rational. The firm is assumed already to have initiated the process of an IPO, paid all filing fees, etc.

All participants first see a public signal y of an exogenous stochastic state $\theta \sim N\left(y, \tau^2\right)$.[7] The state θ can be thought of as the unobserved quality of the firm's project. We regard the public signal y as being revealed during the firm's road show. On seeing y, the firm decides whether to go forward with the IPO or to withdraw it.

If the firm goes forward and raises k units of capital in the IPO, its final value is $e^{\theta} f\left(k\right)$ where f is a differentiable and strictly increasing function that satisfies

$$\iota \stackrel{d}{=} f\left(0\right) > 0 \text{ and } \Omega \stackrel{d}{=} \max_{k \in [0,m]} \frac{f'\left(k\right)}{f\left(k\right)} \in \left(0, \infty\right). \tag{3.1}$$

If the firm withdraws the IPO, its final value is $e^{\theta} f\left(c\right)$ where $c > 0$ is a known constant. Interpreting c literally, it equals the fixed cost of carrying out the IPO versus withdrawing it. However it can also capture the equivalent, in terms of lost capital, of the damage from an IPO that spectacularly fails versus one that is quietly withdrawn in the face of of "adverse market conditions".

Assume henceforth that the firm decides to go forward with the IPO. It then announces a number $s \in [0, 1]$ of shares that are offered for sale, as well as a price $p \geq 0$ per share. Rather than working with s directly, it is more convenient to assume the firm chooses a price p and a capital target $t = ps \in [0, p]$; the number of shares s is then given by t/p. We will assume, without loss of generality, that the capital target t does not exceed the aggregate capital m of the agents as the firm cannot raise more than m units of capital.

After the firm announces p and t, each agent $i \in [0, m]$ then sees a private signal $x_i = \theta + \varepsilon_i$ of the state θ, where $\varepsilon_i \sim N\left(0, \sigma^2\right)$, $\sigma > 0$ is a scalar, and θ and the ε_i's are all

[7] This can be obtained most simply if θ is uniformly distributed on the whole real line and y equals the public signal $\theta + \nu$ where $\nu \sim N\left(0, \tau^2\right)$ is independent of θ. In section 3.3.1.1, we present an alternative derivation in which the prior distribution of θ is normal.

mutually independent. The agents then decide simultaneously whether or not to subscribe: to offer to buy up to $1/p$ shares at the price p.[8] Figure 3.2 shows the time line of the IPO model.

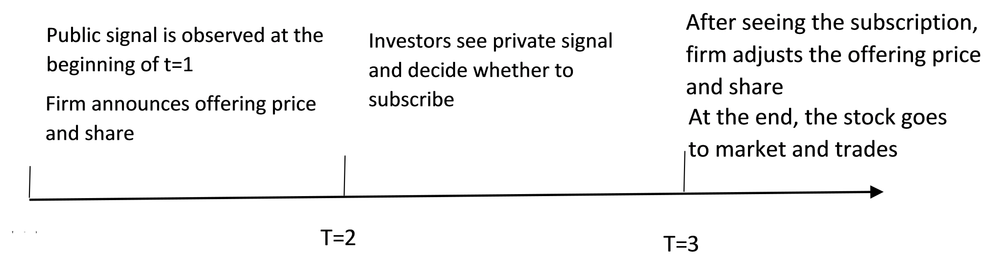

Figure 3.2: Timeline of the IPO Model

Let $\ell \in [0, m]$ be the measure of agents who subscribe or, equivalently, the amount of capital bid by the agents (as each has one unit). If ℓ does not exceed the capital target t, each subscriber transfers her capital to the firm in return for $1/p$ shares. If instead ℓ exceeds t, the IPO is rationed: each subscriber transfers $t/\ell < 1$ units of capital in return in return for $t/(\ell p) < 1/p$ shares while the firm raises t units of capital. An agent's sole alternative investment is a risk-free asset that pays a zero net return. Hence, an agent's net realized payoff from subscribing is

$$\pi_p^t(\theta, \ell) = \begin{cases} \frac{1}{p}\left[e^{\theta} f(\ell) - p\right] & \text{if} \quad \ell \le t \\ \\ \frac{t}{p\ell}\left[e^{\theta} f(t) - p\right] & \text{if} \quad \ell \in [t, m] \end{cases} \tag{3.2}$$

while the firm's realized payoff is

$$\Pi_p^t(\theta, \ell) = \left(1 - \frac{\min\{t, \ell\}}{p}\right) e^{\theta} f(\min\{t, \ell\}).$$

[8] An agent cannot offer to buy more shares since she has only one unit of capital to invest. We say "offer to buy" because demand for shares can exceed supply, in which case the IPO is rationed. (See below.)

An implication is that the firm's maximum payoff from a capital target $t < c$ is less than its payoff, $e^{\theta} f(c)$, from withdrawing the IPO. Thus, if the firm carries out the IPO, it will choose a capital target

$$t \in [c, m]. \tag{3.3}$$

The following standard result from probability theory will be used without proof.

Proposition 3.1. *Suppose we have a variable $\theta \sim N(y, V_\theta)$ to estimate. We observe the variables $x_j = \theta + \varepsilon_j$ for $j = 1, ..., J$, where each $\varepsilon_j \sim N(0, V_j)$ is independent of every $\varepsilon_{j'}$ and of θ. Define the precision of variable j to be $w_j = 1/V_j$. Define $x_0 = y$, $V_0 = V_\theta$, and $w_0 = 1/V_0$. Then the posterior distribution of θ is*

$$\theta^{posterior} \sim N\left(\frac{\sum_{j=0}^{J} w_j x_j}{\sum_{j=0}^{J} w_j}, \frac{1}{\sum_{j=0}^{J} w_j}\right) \tag{3.4}$$

By Proposition 3.1, conditional on the public signal y and the private signal x_i, the state θ is normal with mean

$$\overline{\theta}_{x_i} = \frac{\frac{y}{\tau^2} + \frac{x_i}{\sigma^2}}{\tau^{-2} + \sigma^{-2}} = \frac{\sigma^2 y + \tau^2 x_i}{\sigma^2 + \tau^2} \tag{3.5}$$

and variance S^2 where

$$S = \frac{\sigma\tau}{\sqrt{\sigma^2 + \tau^2}}. \tag{3.6}$$

A threshold equilibrium is one in which an agent i invests if and only if her posterior mean $\overline{\theta}_{x_i}$ is not less than some threshold κ, which may depend on the public signal y and the firm's choices t and p.[9] In such an equilibrium, the measure who invest for given θ and κ is

$$\begin{aligned} \ell &= \ell_{\theta,y}^{\kappa} \overset{d}{=} m \Pr\left(\overline{\theta}_{x_j} \geq \kappa | \theta\right) = m \Pr\left(\overline{\theta}_{x_j} \geq \kappa | \theta\right) \\ &= m \Pr\left(\frac{\varepsilon_j}{\sigma} \geq \frac{\sigma^2(\kappa - y) + \tau^2(\kappa - \theta)}{\tau^2 \sigma} \middle| \theta\right) \\ &= m\left[1 - \Phi\left(\frac{\sigma^2(\kappa - y) + \tau^2(\kappa - \theta)}{\tau^2\sigma}\right)\right] \end{aligned} \tag{3.7}$$

[9]Below we give sufficient conditions for the existence of a unique threshold equilibrium.

by the law of large numbers. Hence, if an agent has posterior mean $\overline{\theta}$ and thinks that each other agent uses the threshold κ, her relative payoff from subscribing is

$$\pi_p^{t*}\left(\overline{\theta},\kappa\right)=\int_{\theta=-\infty}^{+\infty}\pi_p^t\left(\theta,\ell_{\theta,y}^{\kappa}\right)d\Phi\left(\frac{\theta-\overline{\theta}}{S}\right). \tag{3.8}$$

We will assume two conditions that jointly imply the existence of a unique threshold equilibrium. First, the private noise σ is not too small:

$$\sigma>h\left(\Phi^{-1}\left(1-\frac{c}{m}\right)\right) \tag{3.9}$$

where $h\left(z\right)=\frac{\Phi'(z)}{1-\Phi(z)}$ denotes the standard normal hazard function. Second, the public noise is not too small relative to the private noise:

$$\frac{\tau^2}{\sigma}>\max\left\{\frac{m\Omega}{\sqrt{2\pi}},h\left(\Phi^{-1}\left(1-\frac{c}{m}\right)\right)\right\}. \tag{3.10}$$

Our main result is as follows. It shows that there is a unique threshold equilibrium where agents will only invest if their posterior judgement for the mean state $\overline{\theta}_{x_i}$ exceeds the threshold at which the relative payoff from subscribing is zero. Intuitively, if an agent gets a high private signal which indicates that the state is good, he will then expect a good performance of the firm. From his point of view, he believes that other agents also tend to observe a high private signal which encourage them to expect a good state. Since good state means higher firm value, agents are more likely to subscribe. So, in this way, given other agents are adopting threshold strategies (investing if they believes that the posterior state mean is higher than a threshold), an agent will also adopt the threshold strategy to get a positive expected payoff if he has a high enough posterior mean state. At the threshold, the expected payoff should be zero: if the expected payoff is negative, agents will not subscribe; if the expected payoff is positive, agents are then willing to subscribe at a lower posterior mean state.

Theorem 3.1. *Assume (3.9) and (3.10). For any choices p and t of the firm, the agents have a unique threshold equilibrium, where the subscription threshold κ is the unique solution to $\pi_{p,y}^{t*}\left(\kappa,\kappa\right)=0$.*

Proof. Follows directly from Claims 3.2, 3.3, and 3.4. See Appendix. □

3.3 The Simulation

In this section, we will do a simulation to the theoretical model and compare the simulation result to the empirical data.

3.3.1 Preliminaries

We now show how to simulate the large-noise model. We begin with some preliminaries.

3.3.1.1 Making y Stochastic

We have assumed that the state θ is normal with constant mean y. Hence the firm will (for generic parameters) have a unique optimal IPO price p. In order to obtain a *distribution* of IPO prices (and thus of price revisions), the mean y must instead be stochastic. We accomplish this by modelling the prior distribution $N\left(y, \tau^2\right)$ as itself a posterior distribution that results from seeing a public signal of θ which can be interpreted as information that arises from the road show. The mean y then varies with the realization of this public signal. In short, we will be able to assume that y is normal with zero mean and an arbitrary variance $V > 0$ and that, conditional on y, the state θ has the distribution $N\left(y, \tau^2\right)$ where τ must be chosen to satisfy (3.10).

To see why, let us now suppose that the true prior distribution of the state θ is $N\left(0, V_\theta\right)$.[10] Before anyone acts, all participants see a public signal $Z = \theta + \eta$ of the state, where the signal noise $\eta \sim N\left(0, V_\eta\right)$ is independent of θ. By the usual formula for the sum of two independent normal variables, unconditional on θ, the public signal Z is normal with mean $E\left[Z\right] = 0$ and variance $Var\left(Z\right) = V_\theta + V_\eta$. And by Proposition 3.1, given Z, the state θ is normally distributed with posterior mean $y = E\left[\theta|Z\right] = \frac{0/V_\theta + Z/V_\eta}{1/V_\theta + 1/V_\eta} = \frac{V_\theta}{V_\theta + V_\eta}Z$

[10] The zero mean is a normalization: if the mean μ is nonzero, we can replace θ and the firm value function $f\left(\right)$ with $\theta - \mu$ and $e^\mu f\left(\right)$, respectively.

and variance

$$\tau^2 = Var\left(\theta|Z\right) = \frac{1}{1/V_\theta + 1/V_\eta} = \frac{V_\theta V_\eta}{V_\theta + V_\eta}. \tag{3.11}$$

Moreover, when viewed as a random variable (as it is a function of the random variable Z), the posterior expected value $y = E[\theta|Z]$ of the state θ is itself normally distributed with mean

$$E[y] = E[E[\theta|Z]] = \frac{V_\theta}{V_\theta + V_\eta} E[Z] = 0$$

and variance

$$V = Var\left(E[\theta|Z]\right) = Var\left(\frac{V_\theta}{V_\theta + V_\eta} Z\right) = \left(\frac{V_\theta}{V_\theta + V_\eta}\right)^2 Var\left(Z\right) = \frac{V_\theta^2}{V_\theta + V_\eta}. \tag{3.12}$$

3.3.1.2 Bounds on p and t

For each realization y, the we must compute the firm's optimal price p and capital target t. The simplest (but not most efficient) way is by grid search. However, the grid must be finite. Thus, we require upper and lower bounds on each variable. The bounds on t are simple: t must lie in $[c, m]$. And conditional on t, the price p cannot be less than t; else the number $s = t/p$ of shares will exceed one.

It remains to compute an upper bound on p. The idea of the bound is that if the price is too high, the IPO will raise little capital with high probability, so the IPO is not worth its cost c. The bound $\overline{p}_y$, which is increasing in the public signal y, is as follows.

Claim 3.1. *Given a parameter y, a firm that does an IPO will never choose a price p that exceeds the bound*

$$\overline{p}_y = f\left(m\right)\left(\frac{\tau}{\sqrt{2\pi}}\frac{f\left(m\right) - f\left(c\right)}{f\left(c/2\right) - f\left(c\right)}\right)^{\frac{\tau^3}{\sigma^2+\tau^2}} \exp\left(\begin{array}{c} y + \frac{\tau^3}{2}\frac{\tau^2+\tau+1}{\sigma^2+\tau^2} + \frac{S^2}{2} \\ \\ + \frac{\sigma\tau^2}{\sigma^2+\tau^2}\Phi^{-1}\left(1 - \frac{c}{2m}\right)\end{array}\right). \tag{3.13}$$

Proof of Claim 3.1. See Appendix.

3.3.2 Methodology for the Simulation

We now show how to simulate the model. The procedure is thus as follows. One first chooses parameters $m > 0$, $c \in (0, m)$, and $V > 0$. One then chooses a function f; for simplicity, we restrict to the two-parameter family $f(k) = (a+k)^b$ where $a, b > 0$. Once a and b are selected, equation (3.1) pins down the parameters $\iota = a^b$ and $\Omega = \max_{k\in[0,m]} \frac{f'(k)}{f(k)} = \max_{k\in[0,m]} \frac{b(a+k)^{b-1}}{(a+k)^b} = \frac{b}{a}$. Finally, one chooses parameters σ and τ satisfying (3.9) and (3.10). There thus are seven parameters: $(m, c, V, a, b, \sigma, \tau)$.

In order to draw realizations y from the distribution $N(0, V)$ we fix some large positive n and, for each $i = 1, ..., n-1$, let $y_i = V\Phi^{-1}(i/n)$. As this implies $\Phi(y_i/V) = i/n$, each y_i is that y which occurs at exactly the (i/n)th percentile in the distribution $N(0, V)$. One thus can treat each y_i as occurring with equal probability $(n-1)^{-1}$. That is, letting $y_i^n = y_i$,

$$\Phi\left(\frac{y_{i+1}^n}{V}\right) - \Phi\left(\frac{y_i^n}{V}\right) = \frac{i+1}{n} - \frac{i}{n} = \frac{1}{n},$$

whence for any integrable function $g(y)$,

$$\begin{aligned}\int_{y=-\infty}^{\infty} g(y)\, d\Phi\left(\frac{y}{V}\right) &= \lim_{n\to\infty} \sum_{i=1}^{n-1} g(y_i^n)\left[\Phi\left(\frac{y_{i+1}^n}{V}\right) - \Phi\left(\frac{y_i^n}{V}\right)\right] \\ &= \lim_{n\to\infty}\left[\frac{1}{n}\sum_{i=1}^{n-1} g(y_i^n)\right].\end{aligned}$$

For each y_i in $(y_i)_{i=1}^{n-1}$, one then computes the equilibrium threshold κ_{p,y_i}^t (the threshold κ that satisfies $\pi_{p,y_i}^{t*}(\kappa,\kappa) = 0$)[11] for each target $t \in [c, m]$ and price $p \in \left[t, \overline{p}_{y_i}\right]$ in a fine grid. One then searches this grid for the target t and price p that jointly maximize the IPO payoff

$$\Pi(t, p, y_i) = \int_{\theta=-\infty}^{+\infty} \left(1 - \frac{\ell_{p,y_i}^t(\theta) \wedge t}{p}\right) e^{\theta} f\left(\ell_{p,y_i}^t(\theta) \wedge t\right) d\Phi\left(\frac{\theta - y_i}{\tau}\right) \tag{3.14}$$

where "$\wedge$" denotes the pairwise minimum and $\ell_{p,y}^t(\theta)$ denotes the subscription rate $\ell_{\theta,y}^{\kappa_{p,y}^t}$ that results from the equilibrium threshold $\kappa_{p,y}^t$ when the state is θ, the public signal is y, and

[11] By (3.15) and (3.16), κ lies in $\left[\ln\left(\frac{p}{f(m)}\right) - \frac{S^2}{2}, \ln\left(\frac{pm}{c\iota}\right) - \frac{S^2}{2}\right]$. Hence, to find $\kappa_{p,y}^t$ one can perform a bisection search on this finite interval.

the firm's choices are (p, t).[12]. Let us denote the optimal choices as t_i and p_i, and let $\Pi(y_i)$ denote the firm's maximized IPO payoff $\Pi(t_i, p_i, y_i)$. Let I be the set of indices i for which the firm's optimal IPO payoff $\Pi(y_i)$ exceeds its payoff $\Pi_0(y_i) = \int_{\theta=-\infty}^{+\infty} e^{\theta} f(c)\, d\Phi\left(\frac{\theta-y_i}{\tau}\right)$ from withdrawing the IPO. As the firm will carry out the IPO if and only if i lies in I, computed moments should thus be restricted to i in I.

For each i in I, the distribution of the state θ is $N\left(y_i, \tau^2\right)$. We simulate this distribution as follows: for each $j = 1, ..., n-1$, we let $\theta_i^j = y_i + \tau\Phi^{-1}(j/n)$, whence $\Phi\left(\frac{\theta_i^j - y_i}{\tau}\right) = \frac{j}{n}$ so that θ_i^j is the realization that is at the (j/n)th percentile of a $N\left(y_i, \tau^2\right)$ random variable. For each y_i, we thus assign each θ_i^j the same probability weight $(n-1)^{-1}$. For each θ_i^j, the final (end of trading day) value of a share is $p_i^j \overset{d}{=} e^{\theta_i^j} f\left(\ell_i^j \wedge t_i\right)$ where $\wedge$ denotes pairwise minimum and ℓ_i^j denotes the subscription rate $\ell_{\theta,y}^{\kappa}$ that arises from the parameters $\theta = \theta_i^j$ and $y = y_i$ and the subscription threshold $\kappa_i = k_{p_i, y_i}^{t_i}$.[13]

Finally, in order to compute price revisions we require an initial filing price p_0, which is chosen prior to observing y. We will assume for simplicity that the filing price is chosen to minimize the mean squared pricing error $\frac{1}{|I|}\sum_{i\in I}\left(p_{y_i} - p_0\right)^2$ conditional on the IPO going through.[14] Hence, p_0 is computed as the mean $\frac{1}{|I|}\sum_{i\in I} p_{y_i}$ of the final IPO prices p_{y_i} over all public signals y_i for which the firm chooses to carry out IPO. The quantities of interest are then computed as follows for each pair (i, j) such that i is in I:

1. Price Revision: $R_i = \frac{p_i - p_0}{p_0}$.
2. Underpricing: $U_i^j = \frac{p_i^j - p_i}{p_i}$.
3. Oversubscription: $O_i^j = \ell_i^j / t_i$.

[12] The function $\ell_{\theta,y}^{\kappa}$ is defined in (3.7).

[13] A formula for $\ell_{\theta,y}^{\kappa}$ appears in equation (3.7).

[14] It seems reasonable that large price revisions have a reputation cost for the firm's underwriter. If this cost is quadratic, we obtain the given formula for p_0.

3.3.3 The Sample Data

The sample we would like to study consists of firms completing an initial public offering between January 2007 to December 2015 in United States and India. In United States, the data of subscription in IPO are not available as in many other countries. Fortunately, we have found that the data of subscription are publicly available in India. So, we add the data of India to our study. The data of United States comes from Thomson Financial's Securities Data Company (SDC) database. The data of India comes from National Stock Exchange (NSE), Bombay Stock Exchange (BSE) and Chittorgarh Infotech, a company which specialized in providing financial information in India[15]. We exclude unit offers, closed-end funds (including REITs), financial institutions , ADRs of companies already listed in their home countries, limited partnerships, and penny stocks (IPOs with offer prices below five dollars). In addition, we only consider the native companies which is different from most former empirical studies.

A brief description of the data is in Table 3.1. In the sample, there are 935 IPOs for United States and 297 IPOs for India. We can see that the mean and median of initial return for US and India are quite close and they are quite larger than 0 (about 16%- 17%), which suggests the existence of underpricing in the IPOs in both United States and India. The means of initial return are much higher than the medians, which suggests that the distributions skew to the right. The mean of price revision is negative in US, while is positive in India. But both are relatively small in absolute value. Besidesthe medians of price revision are quite near 0 for both countries. For the oversubscription variable in India, we can see that the IPOs in India are generally oversubscribed (most of the oversubscription values are greater than 1).

Table 3.2 describes the correlations between the key variables in US and India. We can

[15]The data are collected from the following websites. National Stock Exchange(NSE): https://www.nseindia.com/products/content/equities/ipos/historical_ipo.htm, Bombay Stock Exchange(BSE): https://www.bseindia.com/markets/PublicIssues/IPOIssues_new.aspx?expandable=3&id=2&Type=P and Chittorgarh Infotech: http://www.chittorgarh.com/ipo/reports/ipo_report_listing_day_gain.asp

see that the IPOs in both US and India tend to display the characteristic of underreaction: the price revision is positively correlated with underpricing. This is one of the most important results indicated from our theoretical model and it is also consistent with previous studies. Intuitively, good news about the state variable lead firms to raise IPO price, which induces a higher price revision. Since agents' wealth is limited, the risk of undersubscription is now greater: investors face greater strategic risk. This leads to a higher underpricing, which appears in the data as underreaction. Also, we can see that underpricing is positively correlated with oversubscription rate.

Table 3.1: Description of Sample data

Underpricing			Price Revision			Oversubscription	
	US	India		US	India		India
Mean	0.161	0.1683	Mean	-0.0287	0.0263	Mean	19.5102
Standard Error	0.009	0.0244	Standard Error	0.0051	0.0035	Standard Error	1.6943
Median	0.0833	0.086	Median	0	0.037	Median	4.51
Mode	0	0.0735	Mode	0	0.0909	Mode	1.11
Standard Deviation	0.2766	0.4197	Standard Deviation	0.156	0.0611	Standard Deviation	29.1998
Sample Variance	0.0765	0.1761	Sample Variance	0.0243	0.0037	Sample Variance	852.6258
Kurtosis	8.9803	4.4852	Kurtosis	0.4542	38.0987	Kurtosis	4.6685
Skewness	2.3456	1.5728	Skewness	-0.4775	-4.6714	Skewness	2.1224
Range	2.5664	3.1067	Range	1.1045	0.6616	Range	160.12
Minimum	-0.3964	-0.6892	Minimum	-0.65	-0.5707	Minimum	0.44
Maximum	2.17	2.4175	Maximum	0.4545	0.0909	Maximum	160.56
Sum	150.4957	49.9854	Sum	-26.7969	7.8111	Sum	5794.515
Count	935	297	Count	935	297	Count	297

3.3.4 Simulation Results

Based on the theoretical model, the simulation data are generated by setting default parameters value as follows: σ=1; $\tau = 1; m = 2; a = 1; b = 1.1; c = 1; \alpha = 1$. We generate 99 public signals y and 999 economic states θ from their normal distributions which are described in Section 3.3.2. In the simulation, 21 IPOs are conducted. Therefore, we

Table 3.2: Sample Correlations

US			
	Underpricing	*PriceRevision*	
Underpricing	1		
PriceRevision	0.468269	1	
India			
	Underpricing	*Price Revision*	*Oversub*
Underpricing	1		
Price Revision	0.09713	1	
Oversub	0.51064	0.071268	1

have 21*99=20979 data points in the simulation. The key variables in the simulation are described in Table 3.3. We can see that the data display underpricing in the simulation.

Table 3.4 displays the correlations between the key variables in the simulation. We can see that the correlation coefficient between underpricing and price revision is 0.128, which indicates the existance of underreaction. This is consistent with the empirical results in India and US and the value of the simulated correlation coefficient between underpricing and price revision is quite similar to the value in India (0.097). The correlation coefficient between underpricing and oversubscription is 0.6. It shows that there exists a very strong positive relation between underpricing and oversubscription. This is also consistent with the empirical result in India and the values of the simulated correlation coefficient between underpricing and oversubscription are close to that in India. Therefore, our simulation tends to be consistent with the empirical results for the correlations among underpricing, price revision and oversubscription. In this way, we provide a simulation result which generates underpricing and underreaction from the strategic risk of undersubscription, which lends additional support to our hypothesis that underpricing and underreaction are caused by the risk of undersubscription.

Table 3.3: Description of Simulation Data

Price Revision		*Underpricing*		*Oversubscription*	
Mean	0	Mean	0.9546	Mean	0.6656
Std Err	0.0021	Std Err	0.0188	Std Err	0.0019
Median	-0.0994	Median	0.0495	Median	0.7325
Mode	-0.3011	Mode	0	Mode	0.5700
Std Dev	0.3104	Std Dev	2.7258	Std Dev	0.2762
Sample Variance	0.0964	Sample Variance	7.4299	Sample Variance	0.0763
Kurtosis	1.1766	Kurtosis	24.4785	Kurtosis	-0.7322
Skewness	1.3685	Skewness	3.9308	Skewness	-0.6379
Range	1.1873	Range	39.9483	Range	0.9986
Minimum	-0.3025	Minimum	-0.9854	Minimum	0.0014
Maximum	0.8848	Maximum	38.9630	Maximum	1
Sum	0	Sum	20026	Sum	13963
Count	20979	Count	20979	Count	20979

Table 3.4: Correlations in Simulation

	Price Revision	*Underpricing*	*Oversubscription*
Price Revision	1		
Underpricing	0.128378	1	
Oversubscription	0.292573	0.602271	1

3.4 Conclusion

This paper examines how strategic risk among investors can help explain both underpricing and underreaction in initial public offerings (IPOs). The strategic risk we studied comes from the assumption that the post-IPO value of a firm can be higher if the IPO raises more capital for the firm. With this assumption, the value of subscribing depends on the aggregate subscription rate. As this risk is resolved immediately after the IPO, the IPO itself is underpriced. Moreover, since individual investors have limited wealth, a higher offer price raises the risk of undersubscription. Investors respond by demanding a larger discount: the offer price appears to underreact to public news.

In this paper, we first use a theoretical model in a global game setting to display the strategic risk of undersubscription in IPO and show how the undersubscription risk can lead to underpricing and underreaction. Then, we conduct a simulation for the model and compare the simulated results to the empirical results in India and US. The simulation results tend to be consistent with the empirical results, which lends further support for our hypothesis that the strategic risk of undersubscription can be used to explain underpricing and underreaciton.

This paper provides a new insight for understanding the underpricing and underreaction in IPO. Our results suggest that undersubscription risk can be an important concern for investors who plan to participate in IPO. Also, the introduction of the endogeneity of firm value in stock market in our analysis may shed new light on the studies of IPOs. These results can also be useful for policy makers in stock market.

3.5 Appendix: Proofs and Technical Results

We first show that there are dominance regions: that it is strictly dominant (not) to subscribe when a player's posterior mean $\overline{\theta}$ is sufficiently high (low). This result does not assume (3.9) or (3.10).

Claim 3.2. *For p and s satisfying $t = ps > 0$:*

$$\textit{for all } \overline{\theta} < \ln\left(\frac{p}{f(m)}\right) - \frac{S^2}{2},\ \pi_p^{t*}\left(\overline{\theta}, \kappa\right) < 0 \textit{ for any } \kappa; \tag{3.15}$$

$$\textit{for all } \overline{\theta} > \ln\left(\frac{pm}{c\iota}\right) - \frac{S^2}{2},\ \pi_p^{t*}\left(\overline{\theta}, \kappa\right) > 0 \textit{ for any } \kappa. \tag{3.16}$$

Proof. Since $t = ps \geq c > 0$, we must have $p > 0$ and $s > 0$. By (3.2),

$$p * \pi_p^t(\theta, \ell) \in \begin{cases} \left[e^\theta \iota - p, e^\theta f(t) - p\right] & \text{if} \quad \ell \leq t \\ \\ \left[\frac{t}{m}\left[e^\theta f(t) - p\right], e^\theta f(t) - p\right] & \text{if} \quad \ell \in [t, m] \end{cases}$$

Combining these cases and using $t \geq c$ and $f(t) \geq \iota$, we obtain, for all ℓ,

$$\pi_p^t(\theta, \ell) \in \left[\frac{e^\theta}{p}\frac{c\iota}{m} - 1, \frac{e^\theta}{p} f(m) - 1\right]$$

which, using $\int_{\theta=-\infty}^{+\infty} e^\theta d\Phi\left(\frac{\theta - \overline{\theta}}{S}\right) = e^{\overline{\theta} + S^2/2}$ and (3.8), implies

$$\pi_p^{t*}\left(\overline{\theta}, \kappa\right) \in \left[\frac{e^{\overline{\theta} + S^2/2}}{p}\frac{c\iota}{m} - 1, \frac{e^{\overline{\theta} + S^2/2}}{p} f(m) - 1\right],$$

from which the claim immediately follows. □

We next show that if a player believes that others are playing a threshold strategy, then an increase in her posterior mean $\overline{\theta}$ strengthens her own incentive to subscribe.

Claim 3.3. *Assume (3.9) and suppose that some player i believes that each other player j will play threshold strategy with threshold κ (i.e., subscribe if and only if $\overline{\theta}_{x_j} > \kappa$). Then player i's relative payoff $\pi_p^{t*}\left(\overline{\theta}, \kappa\right)$ from subscribing is increasing in her posterior mean $\overline{\theta}$.*

Proof. For any ε, let $\overline{\theta}' = \overline{\theta} + \varepsilon$. By the change of variables $\theta' = \theta - \varepsilon$ (whence $\frac{\theta' - \overline{\theta}}{S} = \frac{\theta - \overline{\theta}'}{S}$), we have

$$\begin{aligned} \pi_p^{t*}\left(\overline{\theta}', \kappa\right) &= \int_{\theta=-\infty}^{+\infty} \pi_p^t\left(\theta, \ell_{\theta,y}^\kappa\right) d\Phi\left(\frac{\theta - \overline{\theta}'}{S}\right) \\ &= \int_{\theta'=-\infty}^{+\infty} \pi_p^t\left(\theta' + \varepsilon, \ell_{\theta+\varepsilon,y}^\kappa\right) d\Phi\left(\frac{\theta' - \overline{\theta}}{S}\right) \end{aligned}$$

and thus, renaming θ' to θ,

$$\pi_p^{t*}\left(\overline{\theta}', \kappa\right) - \pi_p^{t*}\left(\overline{\theta}, \kappa\right) = \int_{\theta=-\infty}^{+\infty} \left[\pi_p^t\left(\theta + \varepsilon, \ell_{\theta+\varepsilon,y}^\kappa\right) - \pi_p^t\left(\theta, \ell_{\theta,y}^\kappa\right)\right] d\Phi\left(\frac{\theta - \overline{\theta}}{S}\right)$$

whence

$$\frac{d}{d\overline{\theta}}\pi_p^{t*}\left(\overline{\theta},\kappa\right) = \int_{\theta=-\infty}^{+\infty}\left[\frac{d}{d\theta}\pi_p^t\left(\theta,\ell_{\theta,y}^{\kappa}\right)\right]d\Phi\left(\frac{\theta-\overline{\theta}}{S}\right). \tag{3.17}$$

We now evaluate the integrand. By (3.2),

$$\frac{d}{d\theta}\pi_p^t\left(\theta,\ell_{\theta,y}^{\kappa}\right) = \begin{cases} \underline{\Gamma}_{p,y}^t\left(\theta,\kappa\right) & \text{if} \quad \ell_{\theta,y}^{\kappa} \le t \\ \\ \overline{\Gamma}_{p,y}^t\left(\theta,\kappa\right) & \text{if} \quad \ell_{\theta,y}^{\kappa} \ge t \end{cases}$$

where $\underline{\Gamma}_{p,y}^t\left(\theta,\kappa\right) = \frac{e^{\theta}}{p}\left[f\left(\ell_{\theta,y}^{\kappa}\right) + f'\left(\ell_{\theta,y}^{\kappa}\right)\frac{\partial}{\partial\theta}\ell_{\theta,y}^{\kappa}\right]$ is positive and

$$\begin{aligned}\overline{\Gamma}_{p,y}^t\left(\theta,\kappa\right) &= \frac{e^{\theta}}{p}\left[\frac{t}{\ell_{\theta,y}^{\kappa}}f\left(t\right) - \frac{t}{\left(\ell_{\theta,y}^{\kappa}\right)^2}\left[f\left(t\right)-e^{-\theta}p\right]\frac{\partial}{\partial\theta}\ell_{\theta,y}^{\kappa}\right] \\ &= \frac{e^{\theta}}{p}\frac{t}{\ell_{\theta,y}^{\kappa}}\left(\frac{f\left(t\right)}{\sigma}\left(\sigma - \frac{\sigma}{\ell_{\theta,y}^{\kappa}}\frac{\partial}{\partial\theta}\ell_{\theta,y}^{\kappa}\right) + e^{-\theta}p\frac{1}{\ell_{\theta,y}^{\kappa}}\frac{\partial}{\partial\theta}\ell_{\theta,y}^{\kappa}\right).\end{aligned}$$

To ensure that $\overline{\Gamma}_{p,y}^t\left(\theta,\kappa\right)$ is also positive, it thus suffices to check that whenever $\ell_{\theta,y}^{\kappa} \ge t$, we have

$$\sigma > \frac{\sigma}{\ell_{\theta,y}^{\kappa}}\frac{\partial}{\partial\theta}\ell_{\theta,y}^{\kappa} = \sigma\frac{m}{\ell_{\theta,y}^{\kappa}}\frac{\partial}{\partial\theta}\left(\frac{\ell_{\theta,y}^{\kappa}}{m}\right) = h\left(z_{\theta,y}^{\kappa}\right) \tag{3.18}$$

by (3.7), where

$$z_{\theta,y}^{\kappa} = \frac{\sigma^2\left(\kappa-y\right)+\tau^2\left(\kappa-\theta\right)}{\tau^2\sigma} = \Phi^{-1}\left(1-\frac{\ell_{\theta,y}^{\kappa}}{m}\right). \tag{3.19}$$

But by (3.7) and since h is increasing, $\ell_{\theta,y}^{\kappa} \ge t$ implies that $h\left(z_{\theta,y}^{\kappa}\right)$ does not exceed $h\left(\Phi^{-1}\left(1-\frac{t}{m}\right)\right)$ which, in turn, is not greater than $h\left(\Phi^{-1}\left(1-\frac{c}{m}\right)\right)$ as $t \ge c$. By (3.9), then, (3.18) holds whenever $\ell_{\theta,y}^{\kappa} \ge t$. □

By Claim 3.3, a finite threshold κ is an equilibrium if and only if a player's relative payoff from subscribing $\pi_p^{t*}\left(\overline{\theta},\kappa\right)$ is zero when her posterior mean $\overline{\theta}$ equals the threshold κ: if and only if

$$\pi_{p,y}^{t*}\left(\kappa,\kappa\right) = \int_{\theta=-\infty}^{+\infty}\pi_p^t\left(\theta,\ell_{\theta,y}^{\kappa}\right)d\Phi\left(\frac{\theta-\kappa}{S}\right) \tag{3.20}$$

equals zero. Claim 3.2 implies that $\pi^{t*}_{p,y}(\kappa,\kappa)$ is positive (negative) for sufficiently high (low) thresholds κ. The next claim states that under (3.10), $\pi^{t*}_{p,y}(\kappa,\kappa)$ is continuous and increasing in κ.

Claim 3.4. *1. $\pi^{t*}_{p,y}(\kappa,\kappa)$ is continuous in κ. 2. Assume (3.10). Then $\pi^{t*}_{p,y}(\kappa,\kappa)$ is strictly increasing in κ wherever $\pi^{t*}_{p,y}(\kappa,\kappa)$ is zero.*

Proof. Part 1. Obvious as $\pi^{t*}_{p,y}(\kappa,\kappa)$ is defined in terms of continuous functions. Part 2. For any ε, let $\kappa'=\kappa+\varepsilon$. By the change of variables $\theta'=\theta-\varepsilon$ (whence $\frac{\theta'-\kappa}{S}=\frac{\theta-\kappa'}{S}$), we have

$$\begin{aligned}\pi^{t*}_{p,y}\left(\kappa',\kappa'\right) &= \int_{\theta=-\infty}^{+\infty}\pi^t_p\left(\theta,\ell^{\kappa'}_{\theta,y}\right)d\Phi\left(\frac{\theta-\kappa'}{S}\right)\\ &= \int_{\theta'=-\infty}^{+\infty}\pi^t_p\left(\theta'+\varepsilon,\ell^{\kappa+\varepsilon}_{\theta'+\varepsilon,y}\right)d\Phi\left(\frac{\theta'-\kappa}{S}\right)\end{aligned}$$

and thus, renaming θ' to θ,

$$\pi^{t*}_{p,y}\left(\kappa',\kappa'\right)-\pi^{t*}_{p,y}\left(\kappa,\kappa\right)=\int_{\theta=-\infty}^{+\infty}\left[\pi^t_p\left(\theta+\varepsilon,\ell^{\kappa+\varepsilon}_{\theta+\varepsilon,y}\right)-\pi^t_p\left(\theta,\ell^{\kappa}_{\theta,y}\right)\right]d\Phi\left(\frac{\theta-\kappa}{S}\right),$$

whence

$$\frac{d}{d\kappa}\pi^{t*}_{p,y}\left(\kappa,\kappa\right)=\int_{\theta=-\infty}^{+\infty}\left[\frac{d}{d\varepsilon}\pi^t_p\left(\theta+\varepsilon,\ell^{\kappa+\varepsilon}_{\theta+\varepsilon,y}\right)\right]_{\varepsilon=0}d\Phi\left(\frac{\theta-\kappa}{S}\right). \tag{3.21}$$

By (3.7), $\frac{1}{\ell^{\kappa}_{\theta,y}}\left[\frac{\partial}{\partial\theta}\ell^{\kappa}_{\theta,y}+\frac{\partial}{\partial\kappa}\ell^{\kappa}_{\theta,y}\right]=-\frac{\sigma}{\tau^2}h\left(z^{\kappa}_{\theta,y}\right)$ where $z^{\kappa}_{\theta,y}$ is defined in (3.19). Hence, by (3.2),

$$\left[\frac{d}{d\varepsilon}\pi^t_p\left(\theta+\varepsilon,\ell^{\kappa+\varepsilon}_{\theta+\varepsilon,y}\right)\right]_{\varepsilon=0}=\begin{cases}\underline{\Lambda}^t_{p,y}\left(\theta,\kappa\right) & \text{if}\quad \ell^{\kappa}_{\theta,y}\le t\\[2em] \overline{\Lambda}^t_{p,y}\left(\theta,\kappa\right) & \text{if}\quad \ell^{\kappa}_{\theta,y}\ge t\end{cases}$$

where $\underline{\Lambda}^t_{p,y}(\theta,\kappa)$ denotes

$$\frac{e^{\theta}f\left(\ell^{\kappa}_{\theta,y}\right)}{p}\left[1-\frac{\sigma}{\tau^2}\frac{f'\left(\ell^{\kappa}_{\theta,y}\right)}{f\left(\ell^{\kappa}_{\theta,y}\right)}h\left(z^{\kappa}_{\theta,y}\right)\ell^{\kappa}_{\theta,y}\right]$$

and $\overline{\Lambda}_{p,y}^{t}(\theta,\kappa)$ denotes

$$\frac{t}{p\ell_{\theta,y}^{\kappa}}\left(e^{\theta}f(t)+\frac{\sigma}{\tau^{2}}h\left(z_{\theta,y}^{\kappa}\right)\left[e^{\theta}f(t)-p\right]\right)$$

. Let $\theta_{t,y}^{\kappa}$ be the state θ at which $\ell_{\theta,y}^{\kappa}=t$. By (3.7), $\ell_{\theta,y}^{\kappa}$ is increasing in θ, so $\theta \gtreqless \theta_{t,y}^{\kappa}$ as $\ell_{\theta,y}^{\kappa} \gtreqless t$. By (3.2) and (3.8), we can write $\pi_{p,y}^{t*}(\kappa,\kappa)$ as a sum $A+B$ where A denotes $\int_{\theta=-\infty}^{\theta_{t,y}^{\kappa}}\pi_{p}^{t}(\theta,\ell_{\theta}^{\kappa})\,d\Phi\left(\frac{\theta-\kappa}{S}\right)$ and B denotes $\int_{\theta=\theta_{t,y}^{\kappa}}^{+\infty}\pi_{p}^{t}(\theta,\ell_{\theta}^{\kappa})\,d\Phi\left(\frac{\theta-\kappa}{S}\right)$. Using (3.21) we can write $\frac{d}{d\kappa}\pi_{p,y}^{t*}(\kappa,\kappa)$ as the sum $A'+B'$ where A' denotes $\int_{\theta:\ell_{\theta,y}^{\kappa}\le t}\underline{\Lambda}_{p,y}^{t}(\theta,\kappa)\,d\Phi\left(\frac{\theta-\kappa}{S}\right)$ and B' denotes $\int_{\theta:\ell_{\theta,y}^{\kappa}\ge t}\overline{\Lambda}_{p,y}^{t}(\theta,\kappa)\,d\Phi\left(\frac{\theta-\kappa}{S}\right)$. Since $\ell_{\theta,y}^{\kappa}$ is increasing in θ by (3.7), it follows from (3.2) that $\pi_{p,y}^{t}\left(\theta,\ell_{\theta,y}^{\kappa}\right)$ is negative (positive) for all states θ below (above) some threshold θ^{*} that depends on t, p, and κ. Hence, if $\pi_{p,y}^{t*}(\kappa,\kappa)$ is zero, then $A<0<B$. To show that $\frac{d}{d\kappa}\pi_{p,y}^{t*}(\kappa,\kappa)$ is positive, it thus suffices to show that $A'>0$ and $B'>B$. For the former inequality, $h(z)\,\ell_{\theta,y}^{\kappa}=m\Phi'\left(z_{\theta,y}^{\kappa}\right)\in\left(0,m/\sqrt{2\pi}\right)$ by (3.19) whence, by (3.1), $1-\frac{\sigma}{\tau^{2}}\frac{f'\left(\ell_{\theta,y}^{\kappa}\right)}{f\left(\ell_{\theta,y}^{\kappa}\right)}h\left(z_{\theta,y}^{\kappa}\right)\ell_{\theta,y}^{\kappa}$ is at least $1-\frac{\sigma\Omega m}{\tau^{2}\sqrt{2\pi}}$. Thus, by (3.10), $\underline{\Lambda}_{p,y}^{t}(\theta,\kappa)$ is positive when $\ell_{\theta,y}^{\kappa}\le t$, whence $A'>0$. As for B', assume $\theta\ge\theta_{t,y}^{\kappa}$ so that $\ell_{\theta,y}^{\kappa}\ge t$. Then $z_{\theta,y}^{\kappa}\le\Phi^{-1}\left(1-\frac{c}{m}\right)$ by (3.3) and (3.19) whence $h\left(z_{\theta,y}^{\kappa}\right)$ does not exceed $h\left(\Phi^{-1}\left(1-\frac{c}{m}\right)\right)$ which, by (3.10), is less than $\frac{\tau^{2}}{\sigma}$. Thus, $\overline{\Lambda}_{p,y}^{t}(\theta,\kappa)$ exceeds $\frac{t}{p\ell_{\theta,y}^{\kappa}}\left[e^{\theta}f(t)-p\right]=\pi_{p}^{t}\left(\theta,\ell_{\theta,y}^{\kappa}\right)$ and so $B'>B$ as claimed. □

Proof of Claim 3.1. We first require the following preliminary result:

Lemma 3.1. *For any $\varepsilon>0$ and real number θ_0 satisfying*

$$\theta_{0}>\varphi_{y}(\varepsilon)\overset{d}{=}\tau\left[\tau^{2}+(\tau+1)\left(\frac{1}{2}+\frac{y}{\tau}\right)-\ln\left(\frac{\sqrt{2\pi}}{\tau}\right)-\ln\varepsilon\right], \tag{3.22}$$

we have

$$E_{\theta}\left[e^{\theta}1_{\theta\ge\theta_{0}}\right]<\varepsilon \tag{3.23}$$

where the expectation is taken under the firm's belief that $\theta\sim N\left(y,\tau^{2}\right)$.

Proof. Using the change of variables $z=\frac{\theta-y}{\tau}$, $dz=(d\theta)/\tau$,

$$E_{\theta}\left[e^{\theta}1_{\theta\ge\theta_{0}}\right]=\int_{\theta=\theta_{0}}^{+\infty}e^{\theta}d\Phi\left(\frac{\theta-y}{\tau}\right)=\frac{\tau e^{y}}{\sqrt{2\pi}}\int_{z=\frac{\theta_{0}-y}{\tau}}^{+\infty}e^{g(z)}dz \tag{3.24}$$

where $g(z) = z\tau - z^2/2$. As g is strictly concave and has a slope of -1 at $z = \tau + 1$, it follows that

$$g(z) \leq g(\tau+1) - (z - \tau - 1) = \tau^2 + (\tau+1)/2 - z$$

for all z. Hence, $\int_{z=\frac{\theta_0 - y}{\tau}}^{+\infty} e^{g(z)} dz \leq \exp\left(\tau^2 + \frac{\tau+1}{2} + \frac{y-\theta_0}{\tau}\right)$ which when substituted into (3.24) yields

$$E_\theta\left[e^\theta 1_{\theta \geq \theta_0}\right] \leq \frac{\tau e^y}{\sqrt{2\pi}} \exp\left(\tau^2 + \frac{\tau+1}{2} + \frac{y-\theta_0}{\tau}\right). \tag{3.25}$$

Finally, the right hand side of (3.25) is less than ε if and only if (3.22) holds. □

For any public signal y, let $\kappa_{p,y}^t$ denote the subscription threshold that results from the firm's choices (p,t): the unique solution to

$$\pi_{p,y}^{t*}\left(\kappa_{p,y}^t, \kappa_{p,y}^t\right) = 0. \tag{3.26}$$

Further, let

$$\ell_{p,y}^t(\theta) = \ell_{\theta,y}^{\kappa_{p,y}^t} \tag{3.27}$$

denote the subscription rate that results from the public signal y and the firm's choices (p,t) when the state is θ.[16] By Claim 3.2, $\kappa_{p,y}^t$ is not less than $\ln p - \ln f(m) - S^2/2$. By (3.7), this implies a bound on the subscription rate at θ given y and the choices (p,t):

$$\ell_{p,y}^t(\theta) \leq m\left[1 - \Phi\left(\frac{\left(\sigma^2+\tau^2\right)\left(\ln p - \ln f(m) - S^2/2\right) - \sigma^2 y - \tau^2\theta}{\tau^2\sigma}\right)\right]. \tag{3.28}$$

As this bound is increasing in the state θ, for any price p the subscription rate $\ell_{p,y}^t(\theta)$ is at most $c/2$ as long as θ does not exceed[17]

$$\theta_{p,y} \triangleq \frac{\left(\sigma^2+\tau^2\right)\left(\ln p - \ln f(m) - S^2/2\right) - \sigma^2 y - \tau^2\sigma\Phi^{-1}\left(1 - \frac{c}{2m}\right)}{\tau^2}. \tag{3.29}$$

Hence, the firm's relative payoff from doing the IPO is strictly less than

$$\begin{aligned}
& \int_{\theta=-\infty}^{\theta_{p,y}} e^\theta\left[f\left(\ell_{p,y}^t(\theta)\right) - f(c)\right] d\Phi\left(\frac{\theta-y}{\tau}\right) \\
& + \int_{\theta=\theta_{p,y}}^{+\infty} e^\theta\left[f\left(\ell_{p,y}^t(\theta)\right) - f(c)\right] d\Phi\left(\frac{\theta-y}{\tau}\right) \\
< \quad & E\left[e^\theta\right]\left[f(c/2) - f(c)\right] + E\left[e^\theta 1_{\theta \geq \theta_{p,y}}\right]\left[f(m) - f(c)\right]
\end{aligned}$$

[16] The function $\ell_{\theta,y}^\kappa$ is defined in (3.7).

[17] The bound in (3.29) is obtained by setting the bound in (3.28) equal to $c/2$ and solving for θ.

where the expectations take y as given and assume $\theta \sim N\left(y, \tau^2\right)$. Under this belief, $E\left[e^{\theta}\right]$ equals $e^{y+\tau^2/2}$. Thus, the firm's relative payoff must be negative as long as

$$E\left[e^{\theta} 1_{\theta \geq \theta_{p,y}}\right] < \frac{f\left(c/2\right) - f\left(c\right)}{f\left(m\right) - f\left(c\right)} e^{y+\tau^2/2}.$$

By Lemma 3.1, this must hold if

$$\begin{aligned}
\theta_{p,y} &> \tau\left[\tau^2 + \left(\tau + 1\right)\left(\frac{1}{2} + \frac{y}{\tau}\right) - \ln\left(\frac{\sqrt{2\pi}}{\tau}\right) - \ln\left(\frac{f\left(c/2\right) - f\left(c\right)}{f\left(m\right) - f\left(c\right)} e^{y+\tau^2/2}\right)\right] \\
&= \frac{\tau^3 + \tau^2 + \tau}{2} + y - \tau \ln\left(\frac{\sqrt{2\pi}}{\tau} \frac{f\left(c/2\right) - f\left(c\right)}{f\left(m\right) - f\left(c\right)}\right)
\end{aligned}$$

which by (3.29) can be transformed to $p > \overline{p}_y$. Q.E.D.

CHAPTER 4. BROKERAGE CHOICE, DUAL AGENCY AND HOUSING MARKET STRENGTH

This study develops a theoretical model supported by empirical evidence examining the relation between brokerage choice and market strength. Our model shows that although internal transactions (where both buyer and seller agents are either the same or work for the same firm) have the potential side benefits of higher commission and lower search costs to an agent, in a strong housing market, most brokerage firms still prefer external transactions because of the greater demand for housing. However, when the market weakens, external demand for housing decreases, and brokerage firms become more willing to engage in internal transactions. This occurs at the expense of lowering the selling price, which speaks to a principal-agent incentive misalignment problem. Our model demonstrates that the housing market has a self-correction mechanism for the principal-agent incentive misalignment problem as the market strengthens. Conversely, when the market weakens, internal transactions increase and prices in the market decline, which can further weaken the market. Hence, the equilibrium brokerage choice creates a self-reinforcing mechanism toward generating more extreme market conditions.

4.1 Introduction

Owner-occupied housing units totaled approximately 27 trillion dollars in 2017Q1, making residential real estate one of the most important asset classes in the United States[1]. Han and Hong (2016) report that over 80% of buyers and sellers employ licensed real estate

[1] See Table B.100 entitled Balance Sheet of Households and Nonprofit Organizations in the Federal Reserves Flow of Funds Report, which can be found at http://www.federalreserve.gov/releases/z1/current/.

agents when transacting homes. An agents primary services can be separated into two basic functions: (1) matching, where a real estate agent assists sellers and buyers in finding a suitable trading partner, and then once a match is made, (2) bargaining, where the agent assists the buyer and/or seller in negotiating the terms and conditions of a purchase/sale agreement (Miceli, Pancak, and Sirmans, 2000).

Many studies have examined the role of agents in the housing market. Some focus on the distortion of agency incentives (Gruber and Owings, 1996; Garmaise and Moskowitz,2004; Mehran and Stulz, 2007; Hendel, Nevo, and Ortalo-Magne, 2009). Others examine social inefficiencies resulting from free entry into the real estate brokerage industry (Hsieh and Moretti, 2003; Barwick and Pathak, 2015). Some use search models to explain agency behavior (Yinger 1981; Arnold 1999). Many focus on how brokerage firms affect the relation between selling price and time on the market (Sirmans, Turnbull and Benjamin 1991; Yavas and Yang 1995; Forgey, Rutherford and Springer 1996; Huang and Palmquist 2001; Knight 2002; Turnbull and Dombrow 2006 and Turnbull, Dombrow and Sirmans 2006).

When transacting residential real estate, it can be the case that the buyer and seller are represented by agents who work at different brokerage firms (Han and Hong 2016). Henceforth, we refer to these as external transactions. When the buyer and seller are represented by different agents who happen to work at the same firm, we refer to this relationship throughout the paper as an internal transaction. Finally, when the buyer and seller are represented by the same agent, we refer to this special case of internal transaction as a dual agent transaction. Figure 4.1 displays the relationship between these transaction types. These three brokerage structures have been the source of many studies. For example, Roskelley (2008) offers explanations for transaction distortions for internal transactions based on misaligned incentives and the countervailing force of reputational capital originally investigated in Shapiro (1982, 1983) and Diamond (1989)[2]. Richard and Phillip (2005) use

[2]Internal transactions are sometimes referred to in the literature as dual agency transactions. However, because terms have historically varied widely, confusion in the current study is avoided by only referring to the three brokerage relationships described in the Introduction.

repeat sale methods to test for the price effect associated with internal transactions.

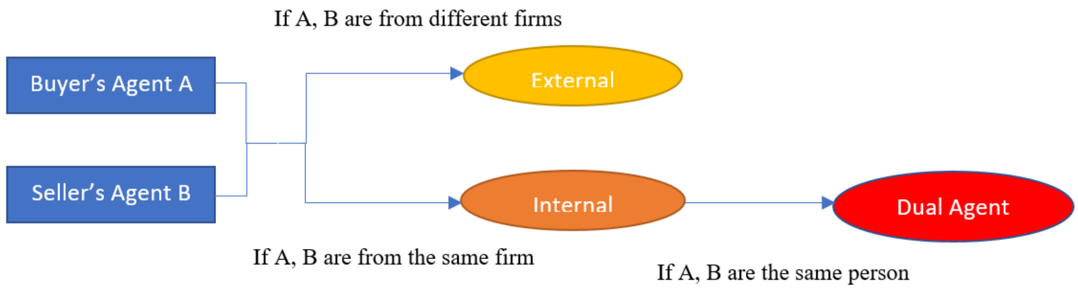

Figure 4.1: Three Types of Transactions Based on Brokerage Structure

In this paper we seek to answer the following questions: (1) When do agents prefer to engage in external versus internal transactions? (2) How do internal transactions, and in particular dual agent transactions, affect sale price? (3) Do these brokerage choices change depending on the strength of the housing market? Our research questions are motivated by the following studies. Gardiner et al. (2007) examine the effect of a law change in Hawaii in 1984 requiring full disclosure of internal transactions and find that internal transactions reduced the sale price, but the effect was much smaller after the legislation (8.0 % versus 1.4 %). Moreover, they also find that internal transactions reduce time on the market by about 8.5% pre-legislation and 8.1% post-legislation. Evans and Kolbe (2005) investigate the effect of internal transactions on price appreciation for houses that are sold twice and find that internal transactions in the first sale have no impact on price appreciation. They also find very limited evidence that an internal transaction in the second sale has a negative effect on price appreciation. Han and Hong (2016) examine to what extent internal transactions are explained by agents strategic incentives as opposed to matching efficiency and find that agents are more likely to promote internal listings when they are financially rewarded. Such effects become weaker when consumers are more aware of agents incentives. Johnson et al. (2015) find that internal transaction distortions on sale price emerge after controlling for the ownership of the property and that such price distortions were different in the periods before and after the financial crisis.

Our study attempts to examine these questions from a new perspective. Specifically, how will the preference for brokerage type change when market strength changes. Moreover, after controlling for market strength, what happens to home prices in internal versus external transactions.

The preference for internal versus external transactions based on relative market strength is motivated by Kadiyali, Prince, and Simon (2011). According to their paper, agents face a variety of incentives and disincentives to engage in behaviors that increase the likelihood of an internal transaction. An internal transaction can be preferred because it allows for a collection of commission on both the buyer and seller side of the ledger. Moreover, an internal transaction may result in a more streamlined closing process allowing the agent to more quickly move onto the next sale. Alternatively, an external sale allows for a potentially much larger buyer pool and therefore a potentially greater selling price and shorter time on the market.

Given the incentives and disincentives provided within the brokerage framework, one may naturally ask how these might change as market conditions change. Motivated by this idea, our study examines how agent preferences for internal transactions change when the market strengthens. To study this question, we first build a theoretical model which shows that when the market gets stronger, firms are more likely to engage in external transactions because the pool of internal buyers and sellers becomes much smaller relative to the external market. Furthermore, our model shows that after controlling for market strength, internal transactions tend to have a lower sale price. The intuition behind this result is that since an internal transaction can capture the commissions from both parties, the agent has a stronger incentive to expedite the transaction by lowering the sale price. To empirically test these relations, we use a detailed set of Multiple Listing Service (MLS) records of single-family transactions in Hampton Roads over the period 1993(Q1) to 2013(Q1), and find that our theoretical results are supported.

The key findings in our paper indicate two important results. First, a potential self-correction mechanism for the principal-agent problem may exist within the housing market. As the market strengthens, external buying orders become more attractive to agents leading them to engage in more external transactions. Note that the principal-agent problem we study here mainly arises from internal transactions. This problem will be reduced by market strength because when the market strengthens, there are fewer internal transactions. Second, our results show that selling prices in internal transactions are lower. So when the market weakens, internal transactions increase. The increase in internal transactions further reduces market price which drives sellers out and further reduces the strength of the market. In this way, the strength of the housing market can reinforce itself through agents' choosing a specific transaction type (internal or external). Hence, the equilibrium brokerage choice creates a self-reinforcing mechanism toward generating more extreme market conditions.

4.2 The Model

Our model is mainly inspired by Yinger (1981), Goetzmann and Peng (2006), Hagiu and Jullien (2011), and Han and Hong (2016). In the model, following Goetzmann and Peng (2006), we assume that the selling agents have full power in deciding whether to sell the house (fully delegation).

The search and match process in the housing market is from Yinger (1981) and Hagiu and Jullien (2011). The search process for buying orders is assumed to follow a Poisson process at rate $\lambda_a^{i,h}$ (search rate), where the search rate is decided by firm i, the house itself h and the order type a ($a = in$ for internal orders and $a = ex$ for external orders). This assumption is consistent with the findings of Bond et al. (2007) in which UK data are used to investigate a number of assumptions associated with the distribution of time on the market. We assume $\lambda_a^{i,h}$ is determined by

$$\lambda_a^{i,h} = k_a^{i,h} N_a$$

where N_{ex} ($a = ex$ for N_a) is the total number of purchase offers that can be searched by an agent in external lists, N_{in} ($a = in$ for N_a) is the total number of purchase offers that can be searched by an agent in internal lists. $k_{in}^{i,h}$, $k_{ex}^{i,h}$ are parameters which depend on firm i and house h. More competent brokerage firms can search faster for buying orders, and better houses can attract buying orders faster. We assume $N_{in} < N_{ex}$, which means that external transactions have larger searching pools (notice that even though we divide the transactions into external and internal pools, in our model the agent will solicit offers in the pooled market). We assume the arrival of internal and external buying orders is independent, so the total process is a combined Poisson process.

$$\lambda^{i,h} = \lambda_{in}^{i,h} + \lambda_{ex}^{i,h} = k_{in}^{i,h} N_{in} + k_{ex}^{i,h} N_{ex}$$

In addition, we assume $k_{in}^{i,h}$, $k_{ex}^{i,h}$ are positive and increasing with firm i's size. Because when a brokerage firm is bigger, it can have more agents and more information for market buying orders. Since $k_{in}^{i,h}$, $k_{ex}^{i,h}$ are positive, a larger searching pool will lead to a higher corresponding search rate $\lambda_a^{i,h}$.

Denote t_j as the waiting time between the arrival of the $(j-1)$-th and the j-th buyer, then the random arrival time of the n-th buyer satisfies

$$T^n = \sum_{j=1}^{n} t_j$$

After waiting for T^n time, the selling agent has received n bids. The selling agent can choose n to set the time he will wait in the market. Denote bid prices as $P_1, P_2, ..., P_n$. Following Cheng, Lin, and Liu (2008), we assume recall is allowed, thus the highest available bidder among the n offer prices is defined as

$$P^n = \max\{P_1, P_2, ..., P_n\}$$

We assume offers are uniformly distributed over the interval $\left[\underline{P}, \bar{P}\right]$. The accepted sale price is X^n, which is the price of the accepted buying order after receiving n offers. The agents choose searching times n in the searching process. During the searching times, internal and external buying orders will arrive in the combined Poisson process. An agent will choose the buying order which gives him the highest commission. let b_a be the commission share for an agent who chooses order type a (recall that a can be either in for an internal transaction or ex for an external transaction), and $b_{in} > b_{ex}$. Her commission is $b_a X^n$.

Assuming the agent is risk neutral, the selling agent's utility depends on his expected payoff and can be represented as

$$U(n, X^n) \triangleq E[b_a X^n - C(n)]$$

where $C(n)$ is the cost function associated with n searches for buying orders. Since the arrival process is assumed to be a combined Poisson Process, we have

$$E(T^n) = \frac{n}{\lambda_{in}^{i,h} + \lambda_{ex}^{i,h}} \tag{4.1}$$

where T^n is the time spent for n searches. The expected cost associated with n searches for buying orders is

$$E(C(n)) = c_T \frac{n}{\lambda_{in}^{i,h} + \lambda_{ex}^{i,h}}$$

where c_T is the per unit time cost for n searches. So the problem becomes

$$\begin{aligned} \underset{n, X^n}{\text{Max}} \qquad & E[b_a X^n] - (c_T \frac{n}{\lambda_{in}^{i,h} + \lambda_{ex}^{i,h}}) \\ \text{subject to} \quad & X^n \leq P^n \end{aligned}$$

In this model, agents will choose the search times n. Then, during the n searches, if the commission for the external buying order of the highest price is higher than the commission for the internal buying orders of the highest price, agents will choose an external order; if the commission for the external buying order of the highest price is lower than the commission for the internal buying orders of the highest price, agents will choose an internal order; if the commission for the external buying order of the highest price is the same as the commission for the external buying order of the highest price, agents will randomize their choice. Figure 4.2 dipicts the tradeoff of the agent between internal and external transaction choices. Thus, we can solve the model in two steps. First, assume n has been decided and use n to find the optimal X^n. Then, substitute X^n into the original problem and find the optimal n.

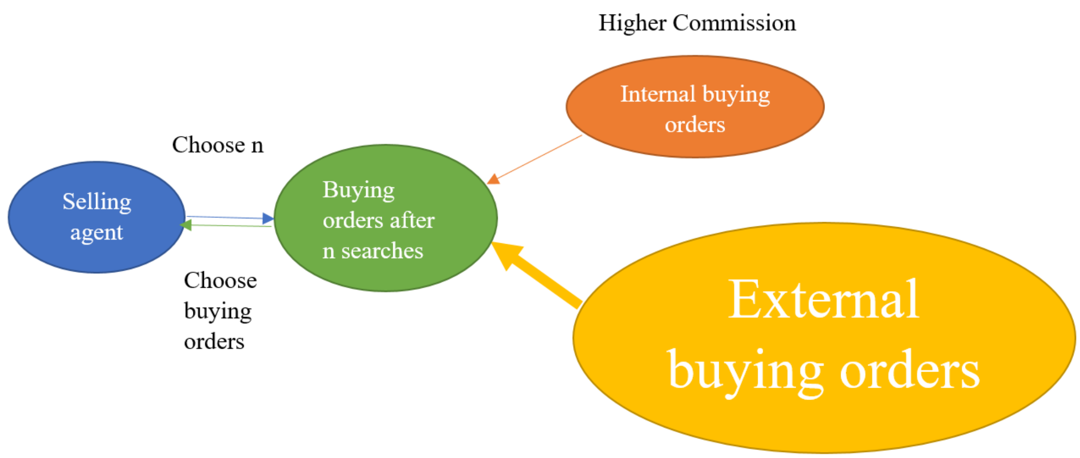

Figure 4.2: Tradeoff Between Internal and External Transactions

In the first step, assuming n is given, the problem is

$$\begin{aligned} &\underset{X^n}{\text{Max}} && E[b_a X^n] - (c_T \frac{n}{\lambda_{in}^{i,h} + \lambda_{ex}^{i,h}}) \\ &\text{subject to} && X^n \leq P^n \end{aligned}$$

It is easy to see that $E(C(n)) \equiv c_T \frac{n}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}}$ is a constant given n. Let p_{in}^n be the probability of accepting an internal buying order after n searches, and let p_{ex}^n be the probability of accepting an external buying order after n searches. Thus, in this model, we have $p_{in}^n + p_{ex}^n = 1$. Denote X_{in}^n as the price of an accepted internal buying order, and denote X_{ex}^n as the price of an accepted external buying order. In addition, let n_a be the number of type a buying orders after n searches and denote P_a^n as the highest price among the searched type a orders. That is to say, after n searches, there will be n_{ex} buying orders from external pool, and the highest price among them is P_{ex}^n; there will be n_{in} buying orders from internal pool, and the highest price among them is P_{in}^n. By definition, we have $n_{in} + n_{ex} = n$. The problem then can be simplified as:

$$\begin{aligned} &\underset{X_{ex}^n, X_{in}^n, p_{in}^n}{\text{Max}} && p_{in}^n b_{in} X_{in}^n + (1 - p_{in}^n) b_{ex} X_{ex}^n - E(C(n)) \\ &\text{subject to} && X_{ex}^n \leq P_{ex}^n, X_{in}^n \leq P_{in}^n, 0 \leq p_{in}^n \leq 1 \end{aligned}$$

Since p_{in}^n, b_{in}, and b_{ex} are all non-negative, it is easy to see that to maximize the expected utility, we have

$$X_{ex}^n = P_{ex}^n, X_{in}^n = P_{in}^n$$

So the problem can be further simplified as

$$\begin{aligned} \underset{p_{in}^n}{\text{Max}} \qquad & b_{ex}P_{ex}^n + p_{in}^n(b_{in}P_{in}^n - b_{ex}P_{ex}^n) - E(C(n)) \\ \text{subject to} \quad & 0 \leq p_{in}^n \leq 1 \end{aligned}$$

Then we can see that if $b_{in}P_{in}^n < b_{ex}P_{ex}^n$, the agent will accept an external buying order, the price of the accepted buying order X^n will be P_{ex}^n; if $b_{in}P_{in}^n > b_{ex}P_{ex}^n$, the agent will accept the internal buying order, and X^n will be P_{in}^n; if $b_{in}P_{in}^n = b_{ex}P_{ex}^n$, the agent will be indifferent, and X^n will be either P_{in}^n or P_{ex}^n. In addition, we assume $b_{in}/b_{ex} < \bar{P}\Big/\underline{P}$, i.e., the commission share gap between an internal and an external transaction should be in a reasonable range, otherwise agents will always choose the internal transaction, which is not consistent with reality.

With this result, the unconditional probability that agents choose internal buying orders becomes

$$\begin{aligned} p_{in} &= Pr(b_{ex}P_{ex}^n \leq b_{in}P_{in}^n) \\ &= \frac{\lambda_{in}^{i,h}}{\lambda_{in}^{i,h} + \frac{\frac{\bar{P}b_{ex}}{b_{in}} - \underline{P}}{\bar{P} - \underline{P}}\lambda_{ex}^{i,h}} \end{aligned} \tag{4.2}$$

And the unconditional probability that agents choose external buying orders is

$$\begin{aligned} p_{ex} &= Pr(b_{ex}P_{ex}^n \geq b_{in}P_{in}^n) \\ &= \frac{\frac{\frac{\bar{P}b_{ex}}{b_{in}} - \underline{P}}{\bar{P} - \underline{P}}\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h} + \frac{\frac{\bar{P}b_{ex}}{b_{in}} - \underline{P}}{\bar{P} - \underline{P}}\lambda_{ex}^{i,h}} \end{aligned} \tag{4.3}$$

To simplify the notation, let

$$\beta \equiv \frac{b_{in}}{b_{ex}}$$

$$\rho \equiv \frac{\frac{\bar{P}}{\beta} - \underline{P}}{\bar{P} - \underline{P}}$$

Thus, substituting the expression of λ_a^i, we have

$$\begin{aligned} p_{in} =& Pr(b_{ex}P_{ex}^n \leq b_{in}P_{in}^n) \\ =& \frac{k_{in}^{i,h}N_{in}}{k_{in}^{i,h}N_{in} + \rho k_{ex}^{i,h}N_{ex}} \end{aligned} \tag{4.4}$$

$$\begin{aligned} p_{ex} =& Pr(b_{ex}P_{ex}^n \geq b_{in}P_{in}^n) \\ =& \frac{\rho k_{ex}^{i,h}N_{ex}}{k_{in}^{i,h}N_{in} + \rho k_{ex}^{i,h}N_{ex}} \end{aligned} \tag{4.5}$$

After solving for X_n, we can substitute it into the original problem and solve for n. In this way, the original problem becomes

$$\begin{aligned} \operatorname*{Max}_n \quad & p_{in}b_{in}E(P_{in}^n | b_{ex}P_{ex}^n \leq b_{in}P_{in}^n) \\ & + p_{ex}b_{ex}E(P_{ex}^n | b_{ex}P_{ex}^n \geq b_{in}P_{in}^n) - E(C(n)) \end{aligned}$$

Since we have

$$E(P_{in}^n | b_{ex}P_{ex}^n \leqslant b_{in}P_{in}^n) = \frac{n\frac{\lambda_{in}^{i,h}+\rho\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}}\bar{P} + \underline{P}}{n\frac{\lambda_{in}^{i,h}+\rho\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}} + 1} \tag{4.6}$$

$$E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n) = \frac{n\frac{1}{\rho}\frac{\lambda_{in}^{i,h}+\rho\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}}\bar{P} + \underline{P}}{n\frac{1}{\rho}\frac{\lambda_{in}^{i,h}+\rho\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}} + 1} \tag{4.7}$$

to simplify the notation, denote

$$\Gamma \equiv \frac{\lambda_{in}^{i,h} + \rho\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h} + \lambda_{ex}^{i,h}}$$

then we can rearrange equations (4.6) & (4.7) as

$$E(P_{in}^n | b_{ex}P_{ex}^n \leqslant b_{in}P_{in}^n) = \frac{n\Gamma\bar{P} + \underline{P}}{n\Gamma + 1}$$

and

$$E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n) = \frac{n\Gamma\bar{P} + \rho\underline{P}}{n\Gamma + \rho}$$

Next substituting the above results, the maximization problem becomes

$$\begin{aligned} \operatorname*{Max}_n \quad & b_{in}\frac{\lambda_{in}^{i,h}}{\lambda_{in}^{i,h} + \rho\lambda_{ex}^{i,h}}\frac{n\Gamma\bar{P} + \underline{P}}{n\Gamma + 1} + b_{ex}\frac{\rho\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h} + \rho\lambda_{ex}^{i,h}}\frac{n\Gamma\bar{P} + \rho\underline{P}}{n\Gamma + \rho} - \\ & (c_T\frac{n}{\lambda_{in}^{i,h} + \lambda_{ex}^{i,h}}) \end{aligned}$$

Taking the derivative with respect to n, we have the First Order Condition (F.O.C) as

$$b_{in}\frac{\lambda_{in}^{i,h}}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}}\frac{\bar{P}-\underline{P}}{(n\Gamma+1)^2}+b_{ex}\frac{\lambda_{ex}^{i,h}}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}}\frac{\bar{P}-\underline{P}}{(n\frac{\Gamma}{\rho}+1)^2}=(c_T)\frac{1}{\lambda_{in}^{i,h}+\lambda_{ex}^{i,h}} \tag{4.8}$$

which can be further simplified as

$$(\bar{P}-\underline{P})(b_{in}\lambda_{in}^{i,h}\frac{1}{(n\Gamma+1)^2}+b_{ex}\lambda_{ex}^{i,h}\frac{1}{(n\frac{\Gamma}{\rho}+1)^2})=c_T \tag{4.9}$$

Next, we start our comparative static analysis with this F.O.C equation.

From equation (4.9), denote $G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)$ as follows

$$G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)\equiv(\bar{P}-\underline{P})(b_{in}\lambda_{in}^{i,h}\frac{1}{(n^*\Gamma+1)^2}+b_{ex}\lambda_{ex}^{i,h}\frac{1}{(n^*\frac{\Gamma}{\rho}+1)^2})-c_T=0$$

where n^* is the optimal n that satisfies the F.O.C.

Taking the derivative of $G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)$ with respect to $\lambda_{ex}^{i,h}$, yields

$$\frac{\partial G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)}{\partial\lambda_{ex}^{i,h}}+\frac{\partial G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)}{\partial(n^*\Gamma)}\frac{\partial(n^*\Gamma)}{\partial\lambda_{ex}^{i,h}}=0$$

Notice that

$$\frac{\partial G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)}{\partial\lambda_{ex}^{i,h}}>0$$

$$\frac{\partial G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)}{\partial(n^*\Gamma)}<0$$

Thus, we have

$$\frac{\partial(n^*\Gamma)}{\partial\lambda_{ex}^{i,h}}=-\frac{\partial G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)}{\partial\lambda_{ex}^{i,h}}\Bigg/\frac{\partial G_{foc}(\lambda_{ex}^{i,h}, n^*\Gamma)}{\partial(n^*\Gamma)}>0$$

Recall that

$$E(P_{in}^n|b_{ex}P_{ex}^n\leqslant b_{in}P_{in}^n)=\frac{n\Gamma\bar{P}+\underline{P}}{n\Gamma+1}$$

then taking the derivative of $E(P_{in}^n|b_{ex}P_{ex}^n\leqslant b_{in}P_{in}^n)$ with respect to $\lambda_{ex}^{i,h}$, becomes

$$\begin{aligned}\frac{\partial E(P_{in}^n|b_{ex}P_{ex}^n\leqslant b_{in}P_{in}^n)}{\partial\lambda_{ex}^{i,h}}&=\frac{\partial(E(P_{in}^n|b_{ex}P_{ex}^n\leqslant b_{in}P_{in}^n)}{\partial(n\Gamma)}\frac{\partial(n\Gamma)}{\partial\lambda_{ex}^{i,h}}\\&=\frac{\bar{P}-\underline{P}}{(n\Gamma+1)^2}\frac{\partial(n\Gamma)}{\partial\lambda_{ex}^{i,h}}>0\end{aligned}$$

Similarly, we can show that

$$\frac{\partial E(P_{in}^n | b_{ex}P_{ex}^n \leqslant b_{in}P_{in}^n)}{\partial \lambda_{in}^{i,h}} > 0$$

Also we have

$$E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n) = \frac{n\Gamma \bar{P} + \rho \underline{P}}{n\Gamma + \rho}$$

then taking the derivative of $E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n)$ with respect to $\lambda_{ex}^{i,h}$, yields

$$\begin{aligned}\frac{\partial E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n)}{\partial \lambda_{ex}^{i,h}} &= \frac{\partial E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n)}{\partial (n\Gamma)} \frac{\partial (n\Gamma)}{\partial \lambda_{ex}^{i,h}} \\ &= \frac{\rho \left(\bar{P} - \underline{P}\right)}{(n\Gamma + \rho)^2} \frac{\partial (n\Gamma)}{\partial \lambda_{ex}^{i,h}} > 0\end{aligned}$$

Similarly, we have

$$\frac{\partial E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n)}{\partial \lambda_{in}^{i,h}} > 0$$

Therefore, as N_{ex} increases, $\lambda_{ex}^{i,h} = k_{ex}^{i,h} N_{ex}$ will increase and lead to an increase in the expected sale price for both internal and external transactions. Also, as N_{in} increases, $\lambda_{in}^{i,h} = k_{in}^{i,h} N_{in}$ will increase and lead to an increase in the expected sale price for both internal and external transactions.

Rewrite equation (4.9), and denote $G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))$ as

$$\begin{aligned}G_{foc}^T(\lambda_{ex}^{i,h}, E(T_{n^*})) \equiv{}& (\bar{P} - \underline{P}) b_{in} \lambda_{in}^{i,h} \frac{1}{\left((\lambda_{in}^{i,h} + \rho\lambda_{ex}^{i,h})E(T_{n^*}) + 1\right)^2} \\ &+ (\bar{P} - \underline{P}) b_{ex} \lambda_{ex}^{i,h} \frac{1}{\left((\lambda_{in}^{i,h} + \rho\lambda_{ex}^{i,h})E(T_{n^*})\frac{1}{\rho} + 1\right)^2} - c_T = 0\end{aligned}$$

where $E(T_{n^*}) = \frac{n^*}{\lambda_{in}^{i,h} + \lambda_{ex}^{i,h}}$ is the expected transaction time.
Taking the derivative of $G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))$ with respect to $\lambda_{ex}^{i,h}$, yields

$$\frac{\partial G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))}{\partial \lambda_{ex}^{i,h}} + \frac{\partial G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))}{\partial (E(T^{n^*}))} \frac{\partial (E(T^{n^*}))}{\partial \lambda_{ex}^{i,h}} = 0$$

Notice that

$$\frac{\partial G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))}{\partial \lambda_{ex}^{i,h}} < 0$$

$$\frac{\partial G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))}{\partial(E(T^{n^*}))} < 0$$

Thus, we have

$$\frac{\partial(E(T^{n^*}))}{\partial\lambda_{ex}^{i,h}} = -\frac{\partial G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))}{\partial\lambda_{ex}^{i,h}} \Bigg/ \frac{\partial G_{foc}^T(\lambda_{ex}^{i,h}, E(T^{n^*}))}{\partial(E(T^{n^*}))} < 0$$

Similarly, we can show that

$$\frac{\partial(E(T^{n^*}))}{\partial\lambda_{in}^{i,h}} =< 0$$

Therefore, as N_{ex} increases, $\lambda_{ex}^{i,h} = k_{ex}^{i,h} N_{ex}$ will go up, and the expected transaction time will decrease. Also, as N_{in} increases, we have similar results.

As the housing market strengthens, there are more external and internal buying orders, i.e. N_{ex}, N_{in} increase. Since there are more new entries in a strong market, external buying orders will increase at a higher rate than internal buying orders, thus $\frac{N_{ex}}{N_{in}}$ also increases. From previous results, we know that the expected sale prices for both internal and external transactions will be higher and the expected transaction time will be shorter as the market strengthens.

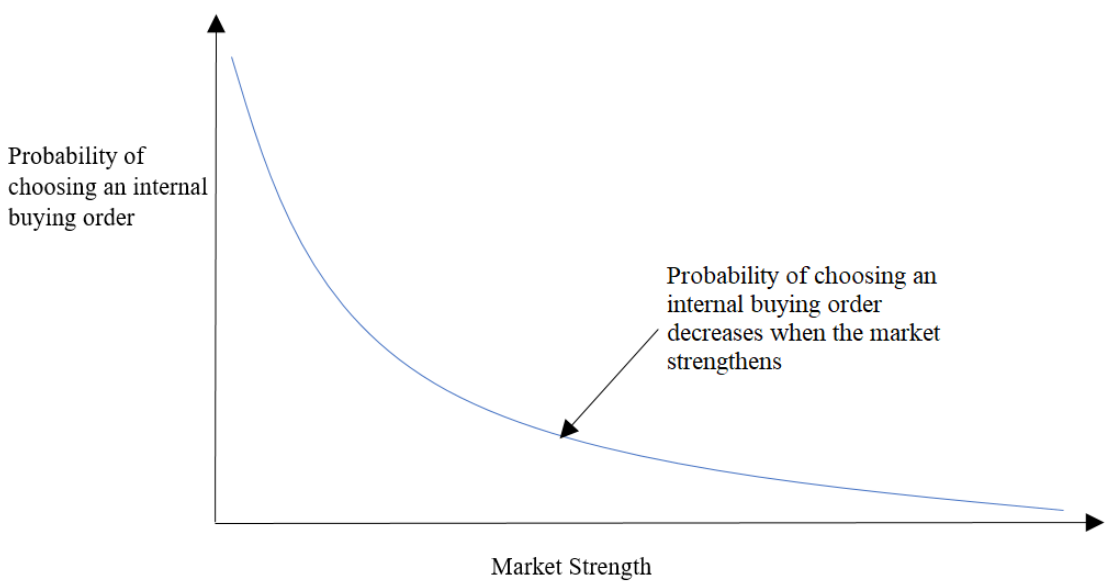

Figure 4.3: The Relation Between Market Strength and Transaction type

Proposition 4.1. *When the market strengthens, i.e., as N_{ex}, N_{in}, $\frac{N_{ex}}{N_{in}}$ increase, the probability of a transaction being internal will decrease, and the probability of a transaction being external will increase.*

This relationship is depicted in Figure 4.3.

Proof. Recall that from equations (4.4) & (4.5), we have

$$p_{in} = Pr(b_{ex}P^n_{ex} \leqslant b_{in}P^n_{in}) = \frac{k^{i,h}_{in}N_{in}}{k^{i,h}_{in}N_{in} + \rho k^{i,h}_{ex}N_{ex}} = \frac{k^{i,h}_{in}}{k^{i,h}_{in} + \rho k^{i,h}_{ex}\frac{N_{ex}}{N_{in}}}$$

and

$$p_{ex} = Pr(b_{ex}P^n_{ex} \geqslant b_{in}P^n_{in}) = \frac{\rho k^{i,h}_{ex}N_{ex}}{k^{i,h}_{in}N_{in} + \rho k^{i,h}_{ex}N_{ex}} = \frac{\rho\frac{N_{ex}}{N_{in}}k^{i,h}_{ex}}{k^{i,h}_{in} + \rho\frac{N_{ex}}{N_{in}}k^{i,h}_{ex}}$$

It is easy to see that

$$\frac{\partial p_{in}}{\partial\frac{N_{ex}}{N_{in}}} < 0$$

and

$$\frac{\partial p_{ex}}{\partial\frac{N_{ex}}{N_{in}}} > 0$$

Thus, when the market strengthens, i.e., when N_{ex}, N_{in}, $\frac{N_{ex}}{N_{in}}$ increase, the probability of an agent choosing internal buying orders, p_{in}, will decrease, and the probability of choosing external buying orders, p_{ex}, will increase. □

Proposition 4.2. *When the search rate ratio between internal and external transactions becomes larger, i.e., when $\lambda^{i,h}_{in}\big/\lambda^{i,h}_{ex}$ increases, the probability of a transaction being internal increases.*

Proof. From equation (4.2), we have

$$P_{in} = \frac{\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}}{\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}} + \frac{\frac{\bar{P}b_{ex}}{b_{in}} - \underline{P}}{\bar{P} - \underline{P}}} \tag{4.10}$$

It is easy to see that P_{in} is increasing in $\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}$.

Rewrite equation (4.9), and denote $G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)$ as

$$G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma) \equiv (\bar{P} - \underline{P})b_{in}\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}\frac{1}{(n^*\Gamma + 1)^2} + (\bar{P} - \underline{P})b_{ex}\frac{1}{(n^*\frac{\Gamma}{\rho} + 1)^2} - \frac{1}{\lambda^{i,h}_{ex}}c_T = 0$$

Then, taking the derivative of $G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)$ with respect to $\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}$, becomes

$$\frac{\partial G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)}{\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}} + \frac{\partial G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)}{\partial (n^*\Gamma)}\frac{\partial (n^*\Gamma)}{\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}} = 0$$

Notice that

$$\frac{\partial G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)}{\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}} > 0$$

$$\frac{\partial G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)}{\partial (n^*\Gamma)} < 0$$

Thus, we have

$$\frac{\partial (n^*\Gamma)}{\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}} = -\frac{\partial G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)}{\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}}\Bigg/\frac{\partial G^{\lambda}_{foc}(\frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}, n^*\Gamma)}{\partial (n^*\Gamma)} > 0$$

□

Proposition 4.3. *The expected sale price of an internal transaction will be less than the expected sale price of an external transaction.*

Proof. Recall that from equations (4.6) & (4.7), we have

$$E(P^n_{in}|b_{ex}P^n_{ex} \leqslant b_{in}P^n_{in}) = \frac{n\Gamma\bar{P} + \underline{P}}{n\Gamma + 1}$$

and

$$E(P^n_{ex}|b_{ex}P^n_{ex} \geqslant b_{in}P^n_{in}) = \frac{n\Gamma\bar{P} + \rho\underline{P}}{n\Gamma + \rho}$$

where

$$\Gamma \equiv \frac{\lambda^{i,h}_{in} + \rho\lambda^{i,h}_{ex}}{\lambda^{i,h}_{in} + \lambda^{i,h}_{ex}}$$

Subtracting the two equations, we have

$$\begin{aligned} E(P_{ex}^n) - E(P_{in}^n) &= \frac{n\Gamma\bar{P} + \rho\underline{P}}{n\Gamma + \rho} - \frac{n\Gamma\bar{P} + \underline{P}}{n\Gamma + 1} \\ &= n\Gamma\frac{\left(\bar{P} - \underline{P}\right)(1-\rho)}{(n\Gamma + \rho)(n\Gamma + 1)} \end{aligned}$$

Notice that

$$\rho \equiv \frac{\frac{\bar{P}}{\beta} - \underline{P}}{\bar{P} - \underline{P}}$$

where

$$\beta \equiv \frac{b_{in}}{b_{ex}} > 1$$

Thus, we have

$$\rho \equiv \frac{\frac{\bar{P}}{\beta} - \underline{P}}{\bar{P} - \underline{P}} < \frac{\bar{P} - \underline{P}}{\bar{P} - \underline{P}} = 1$$

which yields

$$E(P_{ex}^n) - E(P_{in}^n) = n\Gamma\frac{\left(\bar{P} - \underline{P}\right)(1-\rho)}{(n\Gamma + \rho)(n\Gamma + 1)} > 0$$

Therefore, the expected sale price of an internal transaction will be less than the expected sale price of an external transaction. □

Proposition 4.4. *When the search rate ratio between internal and external buying orders becomes larger, i.e., when* $\lambda_{in}^{i,h}\big/\lambda_{ex}^{i,h}$ *increases, the expected sale price for both internal and external transactions will increase.*

Proof. From equations (4.6) & (4.7), we have

$$E(P_{in}^n | b_{ex}P_{ex}^n \leqslant b_{in}P_{in}^n) = \frac{n\Gamma\bar{P} + \underline{P}}{n\Gamma + 1}$$

and

$$E(P_{ex}^n | b_{ex}P_{ex}^n \geqslant b_{in}P_{in}^n) = \frac{n\Gamma\bar{P} + \rho\underline{P}}{n\Gamma + \rho}$$

From previous results, we have

$$\frac{\partial(E(P_{in}^n | b_{ex}P_{ex}^n \leq b_{in}P_{in}^n)}{\partial(n^*\Gamma)} > 0$$

$$\frac{\partial(E(P^n_{ex}|b_{ex}P^n_{ex} \geq b_{in}P^n_{in})}{\partial(n^*\Gamma)} > 0$$

Recall that

$$\Gamma \equiv \frac{\lambda^{i,h}_{in} + \rho\lambda^{i,h}_{ex}}{\lambda^{i,h}_{in} + \lambda^{i,h}_{ex}}$$

which is increasing in $\lambda^{i,h}_{in} \Big/ \lambda^{i,h}_{ex}$

Hence, when $\lambda^{i,h}_{in} \Big/ \lambda^{i,h}_{ex}$ increases, i.e., when the firm's search rate ratio between internal and external buying orders increases, the expected sale price for both internal and external transactions will increase. □

The above results are summarized in Table 4.1. In the Empirical Results section, we will test these theoretical results (Proposition 4.1 to Proposition 4.4) with empirical data.

Table 4.1: Summary of Theoratical Results

Transaction type	Sign
$\partial P^*/\partial N_{ex}$, $\partial P^*/\partial N_{in}$	> 0
$\partial T^*/\partial N_{ex}$, $\partial T^*/\partial N_{in}$	< 0
$\partial p_{in}/\partial \frac{N_{ex}}{N_{in}}$,$\partial p_{ex}/\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}$	< 0
$\partial p_{ex}/\partial \frac{N_{ex}}{N_{in}}$, $\partial p_{in}/\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}$	> 0
$\partial P^*/\partial \frac{\lambda^{i,h}_{in}}{\lambda^{i,h}_{ex}}$	> 0

4.3 Data Description

Our housing transaction data are based upon the complete record of single-family transactions in Hampton Roads over the period 1993(Q1)-2013(Q1), as provided by Real Estate Information Network (REIN). Due to the strength of the data, which includes 375,800 detailed records of housing characteristics including physical structure and neighborhood

Table 4.2: Definition of Variables: Key Variables

Key Variables	Description
Internal Transaction	Equals 1 if the buyer and seller agents work for the same firm; 0 otherwise.
Dual Agent	Equals 1 if the buyer and seller agent is the same person; 0 otherwise.
Price Ratio	The average ratio of sale price to original price during the month immediately preceding the transaction within the same zip code.
Trade Time	The average transaction time during the month before the transaction within the same zip code (in years).
Internal/external Ratio	Ratio of the number of internal transactions to the numbers of external transactions conducted by the brokerage firm within a year of the closed date. This variable serves as a proxy related to the ratio of arrival rates for internal transactions to the rates of external transactions.
Sale price	Selling price of the property (value is in natural log: Log(Sale Price)).
Original Price	Original list price of the property (value is in natural log form: Log(list Price)).

Table 4.3: Definition of Variables: House Characteristics

House Characteristic Variable	Description
#Bathrooms	Number of Bathrooms
#Bedrooms	Number of Bedrooms
#Fireplaces	Number of Fireplaces
#Rooms	Number of Rooms
Square Footage	Size of the house (000s)
#Stories	Number of Stories
Year Built	Years since the home was built (in 10 years)
Tax Amount	Taxes required per year ($ 000s)
#Floors	Number of floors in the home
POAFEE	Extra fees paid to the community to maintain the common elements
Parking	An index ranging from 1 to 4, with 4 being the most desirable parking offered
WaterviewDummy	Equals 1 if home has a water view; 0 otherwise.
CityviewDummy	Equals 1 if home has a city view; 0 otherwise.
WoodsviewDummy	Equals 1 if home has a woods view; 0 otherwise.
WaterDummy	Equals 1 if home is connected to the city water system; 0 otherwise.
AtticDummy	Equals 1 if the home has an attic; 0 otherwise.
FeeSimpleDummy	Equals 1 if the home is owned as fee simple; 0 otherwise.
GasDummy	Equals 1 if water heater is gas; 0 otherwise.
DetachedDummy	Equals 1 if home is detached; 0 otherwise.
NewConstructionDummy	Equals 1 if home is new construction; 0 otherwise.

Table 4.4: Summary of the Key Variables

	(1)				
	count	mean	sd	min	max
Dual Agent	314,977	0.1571	0.3639	0	1
Internal Transaction	314,977	0.2358	0.4245	0	1
Sales Price ($ 000s)	314,977	189.3267	122.28	33	698
Original Price ($ 000s) [a]	314,977	196.2651	129.5172	39.6	750
Price Ratio [b]	314,977	0.9690	0.0701	0.6492	1.1235
Trade Time (year)[c]	314,977	0.1646	0.2037	0.0028	1.0822
Internal/external Ratio[d]	314,977	0.3764	0.2087	0.0119	1.2683

a, *b*, *c*, *d*: To facilitate cross variable comparison, we standardize these variables in our empirical analysis information, we are able to obtain a more accurate estimate of models for internal transactions, expected market price, and time on the market. One major difficulty when examining the price impact due to the impact of brokerage is unobserved housing quality (Shui 2015). To mitigate this problem, we first drop observations that have more than one sale within a year to omit potential housing flippers that might cause changes in house quality. Based on this screen, we jettisoned 60,823 data points, leaving 314,977 observations. Moreover, we take a 99% winsorization of the key variables: sales prices, original list price, price ratio, trade time and internal/external ratio. We then adopt a two-stage process similar to Genesove and Mayer (2001). In stage 1, we first run a full sample hedonic regression with all observable characteristics. We then focus only on repeat sales data and use the residual from the prior transaction of the same unit as a proxy for the unobserved housing quality, and conduct our main analysis in this stage. This treatment leaves 84,238 observations in the second stage analysis. To correct the standard error bias caused by the generated regressor, we use a two-stage bootstrap method for the estimations.

Table 4.2 defines the key variables examined in our model, while Table 4.3 introduces the housing characteristic control variables used in our regression. Table 4.4 provides summary statistics for our key variables. The Dual Agent variable describes whether the transaction is conducted by the same person who works for both sides. From the summary result, we can see that dual agent transactions accounts for 15.71% of all housing market transactions. Similarly, the Internal Transaction variable describes whether the transaction is conducted by the same firm. From the summary result, we can see that internal transactions accounts for 23.58% of all transactions. The average sale price is 189,327, a little lower than the original list price (196,265). Price Ratio describes the ratio of the sale price to the original list price. The mean price ratio in the sample is 0.9690. We use this variable as a proxy of market strength, where a higher ratio suggests a stronger market. Trade Time describes transaction time of a house from listing to selling. The mean trade time in the sample is 0.1646 year (about 2 months). This variable serves as another proxy of market strength in our model, where a shorter trade time suggests a stronger market.

4.4 Empirical Results

Before examining the empirical questions, we present the results from the first stage hedonic regression in Table A4.6.1 (See Appendix).

4.4.1 Impact of Market Strength on Brokerage Choice

To estimate the impact of market strength on brokerage choice, following Han and Hong (2016), we use the following Logistic model:

$$P(d_{ibt} = 1|Z_{it}, X_{it}, W_{bt}) = \frac{\exp(Z_{it}\gamma_e + X_{it}\gamma_i + W_{bt}\delta + \eta_{ibt})}{\exp(Z_{it}\gamma_e + X_{it}\gamma_i + W_{bt}\delta + \eta_{ibt}) + 1}$$

where d_{ibt} is an indicator variable for whether transaction i in period t is an internal transaction carried out by brokerage b, and Z_{it} is a vector of variables measuring market strength. Specifically, $Z_{it} = (PriceRatio_{it}, TradeTime_{it})$, where the $PriceRatio$ is the average ratio of sale price to original list price during the month preceding transaction i within the same zipcode, $TradeTime\ t$ is the average market transaction time during the month before transaction i within the same county. X_{it} refers to a vector of home characteristic control variables including lot size, number of bedrooms, number of bathrooms, a basement dummy, etc. W_{bt} refers to brokerage level variables. Here, we use internal/external transaction ratio as a proxy for W_{bt}. In more detail, the internal/external arrival ratio measures the ratio of the number of internal transactions to the numbers of external transactions conducted by the brokerage firm within a year of the transaction closing date. This variable serves as a proxy related to the ratio of arrival rates for internal transactions to the rates of external transactions. In addition, η_{ibt} contains various fixed effects for the year and month of the transaction, brokerage firm, region, and home characteristics. The estimation of this model is displayed in Tables 4.5 and 4.6. In Table 4.5, internal transactions are reported, whereas in Table 4.6, the more narrowly defined dual agent transaction results are shown.

In comparison to the theoretical model, brokerage firms' differences are represented by their index i; houses' differences are represented by index h; the average time on the market corresponds to the average of $E(T^n(i,h))$; the average sale price on the market corresponds to the average of $E(P^n(i,h))$; Internal/external transaction ratio serves as a proxy for $\frac{k_{in}^{i,h} N_{in}}{k_{ex}^{i,h} N_{ex}}$.

Table 4.5 displays the estimation results for the likelihood of being engaged in an internal transaction. Three different models are estimated. Column 1 is the baseline estimation where market strength is measured using both the ratio of sale price to original list price (i.e., the price premium effect) and market transaction time (i.e., the liquidity premium effect). The related coefficient for price ratio estimated in column 1 is statistically significant, and the sign is consistent with expectations. The ratio of sale price to original list price has a negative impact on the probability of a realized internal transaction. We can also see that market transaction time has a positive impact on the probability of an internal transaction, although the coefficient is not significant. When the market gets stronger, the ratio of sale price to original list price increases, market transaction time decreases, and the probability of an internal transaction decreases. This is consistent with Proposition 4.1 which claims the probability of a transaction being internal will decrease with market strength. From the estimation of the Logistic model, we can observe the average marginal effect of the variables. For example, when other variables are evaluated at their average value, a 1 standard deviation increase in the ratio of sale price to original list price will produce a 1.0106% decrease in the probability of an internal transaction being realized. When other variables are evaluated at their average value, a 1 standard deviation increase in market time will produce a 0.2658% increase in the probability of a realized internal transaction. Compared with the average proportion for an internal transaction (23.58%), this effect is not negligible. Furthermore, note that the empirical estimation is for the realized probability that the transaction is internal. Since the market shares for the new buying orders are different among firms, the willingness to choose external transactions may not be fully

Table 4.5: Impact of Market Strength on Brokerage Choice

	(1)	(2)	(3)	(4)
Internal Transaction				
Price Ratio	-0.0775***	-0.0845***	-0.0845***	-0.0488***
	(0.0170)	(0.0176)	(0.0140)	(0.0164)
Trade Time	0.0193	0.0270	0.0270*	0.0045
	(0.0202)	(0.0199)	(0.0161)	(0.0187)
Internal/external Ratio	1.1341***	0.5113***	0.5113***	0.5264***
	(0.0929)	(0.0602)	(0.0628)	(0.0642)
Stage 1 Residual	-0.0425	-0.0445	-0.0445	-0.0803
	(0.0858)	(0.0847)	(0.0756)	(0.0769)
FEregion	Yes	Yes	Yes	Yes
FEyearmonth	Yes	Yes	Yes	Yes
FEbrokerageoffice	No	Yes	Yes	Yes
FEzipcode*month	No	No	No	Yes
House Characteristics	Yes	Yes	Yes	Yes
Original Price	Yes	Yes	Yes	Yes
Constant	-1.6151***	-0.8591	-0.8591*	13.6790
	(0.2389)	(0.5338)	(0.4855)	(0.9523)
Number of Observation	84,238	84,238	84,238	84,238

Note: Robust standard errors clustered at zip code level in parentheses for (1), (2). Robust standard errors clustered at zip code level and brokerage office level in parentheses for (3), (4).
* Significant at 10% level, ** Significant at 5% level, *** Significant at 1% level

Table 4.6: Impact of Market Strength on Dual Agent Preference

	(1)	(2)	(3)	(4)
Dual Agent				
Price Ratio	-0.0944***	-0.1026***	-0.1026***	-0.0563***
	(0.0249)	(0.0239)	(0.0175)	(0.0208)
Trade Time	0.0329	0.0356	0.0356*	0.0074
	(0.0235)	(0.0229)	(0.0192)	(0.0232)
Internal/external Ratio	0.8725***	0.5278***	0.5278***	0.5480***
	(0.0726)	(0.0649)	(0.0676)	(0.0693)
Stage 1 Residual	-0.1083	-0.0598	-0.0598	-0.0800
	(0.1257)	(0.1195)	(0.1065)	(0.1095)
FEregion	Yes	Yes	Yes	Yes
FEyearmonth	Yes	Yes	Yes	Yes
FEbrokerageoffice	No	Yes	Yes	Yes
FEzipcode*month	No	No	No	Yes
House Characteristics	Yes	Yes	Yes	Yes
Original Price	Yes	Yes	Yes	Yes
Constant	-2.2490***	-1.0575*	-1.0575**	15.1511
	(0.3019)	(0.6007)	(0.5365)	(0.7863)
Number of Observation	84,238	84,238	84,238	84,238

Note: Robust standard errors clustered at zip code level in parentheses for (1), (2). Robust standard errors clustered at zip code level and brokerage office level in parentheses for (3), (4).
* Significant at 10% level, ** Significant at 5% level, *** Significant at 1% level

realized in reality when the market strengthens. So market strength can have a greater impact on the preference for internal transactions than the estimated result. Recall that, from the theoretical section, we have

$$\frac{\partial p_{in}}{\partial \frac{N_{ex}}{N_{in}}} = = - \frac{k_{in}^{i,h}}{(k_{in}^{i,h} + \rho k_{ex}^{i,h} \frac{N_{ex}}{N_{in}})^2} \rho k_{ex}^{i,h} = - \frac{p_{in}^2}{k_{in}^{i,h}} \rho k_{ex}^{i,h}$$

which means the impact of market strength on preference for an internal transaction will be greater for firms with a higher probability of choosing internal transactions. Note that the estimated result is for the average effect, so agents who previously had a higher probability of choosing an internal transaction will be more impacted by market strength. This implies that the estimated effect is stronger for firms who are mainly engaged in internal transactions. If a firm is primarily engaging in internal transactions, our results indicate that market strength may have a huge impact on its preference for choosing the type of transaction.

Concerning the control variables, the coefficient associated with the internal/external transaction ratio in the office is positive and highly significant, which is also consistent with Proposition 4.2 which claims the probability of a transaction being internal will increase with the search rate ratio between internal and external transactions. Intuitively, when the internal/external transaction ratio is larger, a firm's search rate for internal buying orders is higher. Thus, the incentive for internal transactions increases, which leads to more internal transactions.

In the baseline estimation, we control for a wide range of attributes including home characteristics, region, time, and so forth. To control for the potential effect of unobserved brokerage office characteristics, we include brokerage office fixed effects in the baseline model. The result in column 2 reveals that the key coefficient estimates on ratio of sale price to original list price and internal/external transaction ratio continue to be significant and have the expected sign. We can also see that the coefficient on market transaction time remains positive, although the coefficient is not significant. This suggests that the unobserved brokerage office effect is unlikely to change the interpretation of our findings.

To allow for intragroup autocorrelation within the area and the brokerage office, we estimate a model with two-way clustering at both the zip code level and brokerage office level. We can see in column 3 that the signs and significance levels of the price ratio and internal/external transaction ratio remain the same, which indicates that our results are robust to this change. In addition, the coefficient on market transaction time becomes significant. The results presented here demonstrate a strong relation between market strength and the probability of engaging in internal transactions.

To control for interacting effects of region and time, in column 4, we include the interaction term of zip code and the month of closing date. We can see that the key coefficient estimates on ratio of sale price to original list price and internal/external transaction ratio continue to be significant and have the expected sign. This finding lends further support to the robustness of our result.

We next examine the relation between market strength and the probability of engaging in dual agent transactions, a subset of internal transactions where the buyer and seller are represented by the same agent. In Table 4.6, we see that the sign and significance level of the coefficient estimates on ratio of sale price to original list price and market transaction time remain qualitatively similar.

4.4.2 Impact of Internal Transactions on Sale Price

To estimate the impact of an internal transaction on sale price, we use the log-linear hedonic model:

$$\ln P_{ibt} = d_{ibt}\theta_1 + Z_{it}\alpha_1 + X_{it}\beta_1 + W_{bt}\delta_1 + \eta_{1ibt}$$

where T_{ibt} is the market transaction time of transaction i carried out by brokerage b at time t. As before, d_{ibt} is an indicator variable for whether transaction i at period t is an internal transaction carried out by brokerage b. Z_{it} is a vector of variables measuring market strength: $Z_{it} = (PriceRatio_{it}, TradeTime_{it})$. X_{it} refers to a vector of home characteristic control variables. W_{bt} refers to brokerage level variables. As before, we use internal/external transaction ratio as a proxy for W_{bt}. In addition, η_{1ibt} contains various fixed effects. The estimation results are displayed in Tables 4.7 and 4.8. Table 4.7 reflects internal transactions, whereas Table 4.8 reports results for dual agent transactions.

Table 4.7 displays estimation results for sale price using three nested specifications. To control for unobserved home quality which impacts sale price, we include the original list price in all of the three estimations. In column 1, the baseline model, we see that an internal transaction has a negative impact on sale price after controlling for market strength, among other variables. Because the sale price is in log form, we follow Kennedy (1981) and interpret the coefficient of an internal transaction on sale price as

$$g = e^{\widehat{\gamma_{in}} - \frac{1}{2}Var(\widehat{\gamma_{in}})} - 1$$

Using this formula, we obtain $g = -0.0059$. That is, an internal transaction is associated with a 0.59% sale price decrease after controlling for market strength. The reason for this negative relation may be that firms can earn a higher share of the commission for internal transactions, so they are willing to accept a lower sale price. This empirical result is consistent with Proposition 4.3 which claims that the expected sale price of the internal transactions will be less than the expected sale price of the external transactions. As for the effect of market strength, we can see that the price ratio of sale price to original list price is positively correlated with sale price. The coefficient on market transaction time is positive and significant suggesting that although a longer market transaction time indicates lower offers, agents may be willing to wait longer in a colder market, which increases the number of offers received and hence increases the sale price. Moreover, the coefficient on internal/external transaction ratio is positive and significant. Intuitively, when the internal/external transaction ratio is greater, the firm will get a higher proportion of internal buying orders during its search. Since internal transactions have greater commission share, the higher probability of the presence of internal buying orders will encourage firms to search more. Therefore, since there are more buying orders to choose from, the sale price will be higher. This empirical result is consistent with Proposition 4.4 which posits that the

Table 4.7: Impact of Internal Transactions on Sale Price

	(1)	(2)	(3)	(4)
Log(Sale price)				
Internal Transaction	-0.0059**	-0.0086***	-0.0086***	-0.0100***
	(0.0026)	(0.0027)	(0.0017)	(0.0016)
Price Ratio	0.0165**	0.0159***	0.0159***	0.0108***
	(0.0065)	(0.0060)	(0.0013)	(0.0013)
Trade Time	0.0077*	0.0079**	0.0079***	0.0053***
	(0.0040)	(0.0037)	(0.0014)	(0.0012)
Internal/external Ratio	0.0474***	0.0133***	0.0133***	0.0144***
	(0.0077)	(0.0033)	(0.0038)	(0.0037)
Stage 1 Residual	0.1504***	0.1504***	0.1504***	0.1254***
	(0.0208)	(0.0197)	(0.0212)	(0.0185)
FEregion	Yes	Yes	Yes	Yes
FEyearmonth	Yes	Yes	Yes	Yes
FEbrokerageoffice	No	Yes	Yes	Yes
FEzipcode*month	No	No	No	Yes
House Characteristics	Yes	Yes	Yes	Yes
Original Price	Yes	Yes	Yes	Yes
Constant	11.4160***	11.3505***	11.3505***	11.1975***
	(0.0679)	(0.0633)	(0.0603)	(0.0729)
R-Square	0.9136	0.9182	0.9182	0.9234
Number of Observation	84,238	84,238	84,238	84,238

Note: Robust standard errors clustered at zip code level in parentheses for (1), (2). Robust standard errors clustered at zip code level and brokerage office level in parentheses for (3) and (4).
* Significant at 10% level, ** Significant at 5% level, *** Significant at 1% level

Table 4.8: Impact of Dual agent on Sale Price

	(1)	(2)	(3)	(4)
Log(Sale price)				
Dual Agent	-0.0094***	-0.0099***	-0.0099***	-0.0107***
	(0.0033)	(0.0032)	(0.0023)	(0.0021)
Price Ratio	0.0164**	0.0159***	0.0159***	0.0108***
	(0.0065)	(0.0060)	(0.0013)	(0.0013)
Trade Time	0.0077*	0.0079**	0.0079***	0.0053***
	(0.0040)	(0.0037)	(0.0014)	(0.0012)
Internal/external Ratio	0.0474***	0.0132***	0.0132***	0.0143***
	(0.0077)	(0.0033)	(0.0037)	(0.0037)
Stage 1 Residual	0.1503***	0.1504***	0.1504***	0.1254***
	(0.0207)	(0.0197)	(0.0212)	(0.0185)
FEregion	Yes	Yes	Yes	Yes
FEyearmonth	Yes	Yes	Yes	Yes
FEbrokerageoffice	No	Yes	Yes	Yes
FEzipcode*month	No	No	No	Yes
House Characteristics	Yes	Yes	Yes	Yes
Original Price	Yes	Yes	Yes	Yes
Constant	11.4161***	11.3511***	11.3511***	11.1994***
	(0.0679)	(0.0636)	(0.0604)	(0.0729)
R-Square	0.9136	0.9182	0.9182	0.9234
Number of Observation	84,238	84,238	84,238	84,238

Note: Robust standard errors clustered at zip code level in parentheses for (1), (2). Robust standard errors clustered at zip code level and brokerage office level in parentheses for (3) and (4).
* Significant at 10% level, ** Significant at 5% level, *** Significant at 1% level

expected sale price will increase with the search rate ratio between internal and external transactions after controlling for the type of transaction.

To control for the potential effect of unobserved brokerage office characteristics, we include brokerage office fixed effects in the baseline model. The key result in column 2 remains similar to the baseline result. To allow for intragroup autocorrelation within the area and brokerage office, we estimate in column 3, a model in which robust standard errors are clustered at both the brokerage office level and zip code level. The significance levels of the key coefficient estimates remain the same, which speaks to the robustness of the result. As before, we next examine the relation between sale price and dual agent transactions. As reported in Table 4.8, the sign and significance level of the coefficient estimates remain similar. The only difference is that the coefficient on dual agent becomes more negative.

From Tables 4.7 and 4.8, we see that for internal transactions, sale prices will be lower. This result shows the principal-agent incentive misalignment problem in the housing market. Sellers want to sell the house at the highest price, while agents want to earn the highest commissions at a given searching cost. Thus, for internal transactions, agents are willing to accept lower prices offered by internal buying orders to receive higher commission, which is not in the seller's best interest. This incentive misalignment causes the principal-agent problem within the housing market. From Tables 4.5 and 4.6, we see that as the market strengthens, agents are more likely to engage in external buying orders, which helps reduce the principal-agent incentive misalignment problem. This result indicates that the housing market has a self-correction mechanism for the principal-agent problem. As the market strengthens, external buying orders become more attractive causing agents to engage in more external transactions. Since the principal-agent incentive misalignment problem we study here mainly comes from internal transactions, it is mitigated when the market strengthens. From the above results, we have another important implication. Since selling prices in internal transactions are lower, when the market weakens, internal transactions increase and selling prices tend to be further reduced. A low price in the housing market can drive sellers out of the market, further weakening it. When the market strengthens, the opposite situation tends to occur. In this sense, the strength of the housing market can reinforce itself through transaction preference.

4.5 Conclusion

Many studies have been conducted to understand the impact that brokerage representation has on the home transaction process. We investigate brokerage choice not only between external (where agents from different firms represent the buyer and the seller) versus internal (where different agents from the same firm represent the buyer and the seller) transactions, but also for a subset of internal transactions known as dual agent transactions, where a single agent represents both the buyer and the seller in the same transaction.

We begin by building a theoretical model to establish a framework on which an empirical model is based. Consistent with our theory, we find that as the housing market strengthens, brokerage choice shifts to external transactions because the relative demand pool becomes much greater potentially resulting in a higher selling price and shorter time on the market. Moreover, after controlling for market strength, we find that internal transactions result in a lower sale price. The intuition behind this result is that since agents in internal transactions capture higher commissions from both parties, they have a stronger incentive to expedite the transaction at the expense of lowering the sale price. This speaks to the principal-agent problem in residential brokerage.

In sum, different from Johnson et al. (2015), which finds that dual agent brokerage has no effect on sale price, our result suggests that internal transactions tend to lower sale price (which harms the seller). But, when the market gets stronger, there are fewer internal transactions, and this agency problem is mitigated. As such, the housing market has a self-correction mechanism for the principal-agent incentive misalignment problem. In comparison with Han and Hong (2016), which finds that agents are more likely to promote internal listings when they are financially rewarded and that this effect becomes weaker when consumers are more aware of agents' incentives, our study provides another kind of incentive misalignment between real estate agents and their clients, and the potential self-correction mechanism in the market. This result is useful to real estate industry participants in that sellers suffer from a suboptimal selling price. The good news is that as the market strengthens, the principal-agent problem will be mitigated. However, the strength of the housing market can be self-reinforcing. We find that internal transactions are associated with lower transaction prices. So, when the market weakens, the ratio of internal transactions in the market increases and prices decline, which can cause the market to further weaken. Hence, the equilibrium brokerage choice creates a self-reinforcing mechanism toward generating more extreme market conditions.

4.6 Appendix: Hedonic Regression

Table A4.6.1: Hedonic Regression

	Log(Sale price)
#Baths_Full	0.2080***
	(0.0149)
#Baths_Half	0.0873***
	(0.0135)
#Bedrooms	0.0752***
	(0.0072)
#Fireplaces	0.0712***
	(0.0076)
#Rooms	0.0264***
	(0.0025)

	Table A4.6.1 (continued)
Square Footage	0.0030***
	(0.0005)
#Stories	0.0003
	(0.0005)
Year Built	0.0001
	(0.0001)
Tax Amount	0.003***
	(0.0007)
#Floors	0.0931***
	(0.0080)
POAFEE	0.0159
	(0.0199)

	Table A4.6.1 (continued)
Parking	0.115***
	(0.0121)
WaterviewDummy	0.0641***
	(0.0053)
CityviewDummy	0.1860***
	(0.0260)
WoodsviewDummy	0.1900***
	(0.0233)
WaterDummy	0.1240***
	(0.0126)
AtticDummy	0.0473***
	(0.0079)

	Table A4.6.1 (continued)
FeeSimpleDummy	-0.1180***
	(0.0212)
GasDummy	0.0728***
	(0.0080)
DetachedDummy	0.2410***
	(0.0192)
NewConstructionDummy	0.1830***
	(0.0112)
FEregion	Yes
FEyearmonth	Yes
Constant	10.4400***
	(0.0583)

	Table A4.6.1 (continued)
R-Square	0.7570
Number of Observations	314,977

Note: Robust standard errors clustered at zip code level in parentheses.

* Significant at 10% level, ** Significant at 5% level, *** Significant at 1% level

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